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INTERIM PROGRESS REPORT

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ANALYTICAL AND EXPERIMENTAL  
CHARACTERIZATION OF DAMAGE PROCESSES  
IN COMPOSITE LAMINATES

by

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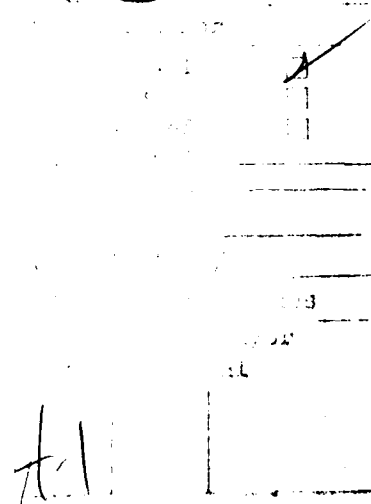
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- (iii) Analysis of distributed damage caused in a composite ply by either transverse crack or fiber breaks. Several methods, such as self-consistent estimates, shear lag approximations, crack array models, and finite element analysis of cracks in an embedded ply were employed. It was found that these methods give very similar predictions of stiffness reductions of plies and laminates, and that these predictions are in good agreement with available experimental data.

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## ABSTRACT

A summary of results is presented on the subject of damage development in metal and polymer matrix composite laminates. The following technical developments are described:

- (i) Evaluation of crack densities, stiffness changes, and fiber stresses caused by cyclic loading in three 6061-0 Al-B laminates, with 0g,  $(0/90)_2$ s, and  $(0/\pm 45/90/0/\pm 45/\frac{1}{2}90)_s$  layers. This problem is solved in an incremental way, with regard for interaction between plastic deformation and matrix crack growth in individual plies. Saturation damage states are predicted at different levels of steady cyclic loading. Good comparison is obtained with available experimental data,
- (ii) Analysis of first ply failure in polymer matrix composites. The influence of ply thickness on strength is predicted in terms of flaw nucleation mechanisms, and
- (iii) Analysis of distributed damage caused in a composite ply by either transverse cracks or fiber breaks. Several methods, such as self-consistent estimates, shear lag approximations, crack array models, and finite element analysis of cracks in an embedded ply were employed. It was found that these methods give very similar predictions of stiffness reductions of plies and laminates, and that these predictions are in good agreement with available experimental data.

## 1. INTRODUCTION

This research project is conducted as a cooperative effort of two investigators. Dr. George J. Dvorak is the Principal Investigator of the program at Rensselaer Polytechnic Institute, which is the primary contractor. Dr. Norman Laws is Principal Investigator of the part of the program subcontracted from RPI to the University of Pittsburgh.

In the first year of this research program, major accomplishments were achieved in the following areas:

- mechanics of fatigue damage in metal matrix laminates (Dvorak)
- mechanics of first ply failure in polymer matrix laminates (Dvorak and Laws)
- mechanics of distributed damage caused in a composite ply by either transverse cracks or fiber breaks, and the effect of the damage on behavior of laminated plates (Laws and Dvorak).

In addition, significant progress has been made in analysis of damage caused by propagation of cracks toward, across, and along ply interfaces in polymer matrix systems.

The principal results are described in the sequel.

## 2. STATUS OF THE RESEARCH

### 2.1 Mechanics of Fatigue Damage in Metal Matrix Composite Laminates

Several years ago, Dvorak and Johnson [1] observed that B-Al laminates may suffer a substantial (~ 50%) reduction of stiffness and strength during cyclic loading well below the endurance limit. A similar effect was found more recently by Johnson [2] in fibrous SiC-Al laminates and it is now apparent that metal matrix systems reinforced by monolayers of large diameter fibers are sensitive to fatigue damage when the matrix is subjected to cyclic plastic straining during fatigue loading. It is not yet clear that similar damage occurs in other MMC, e.g., those reinforced by graphite, FP, or other fibers of very small diameter. In any case, this type of damage is caused by low cycle fatigue cracks which can grow in off-axis plies on planes parallel to the fiber axis. Typically, many aligned slit cracks are found in each ply. They may propagate in the matrix, at the fiber-matrix interface, and, on occasion, a split fiber may be a part of such a crack. Figure 1 shows various examples of these cracks in a  $(0/90/\pm 45)_s$  laminate, which were constructed from observations of actual crack systems described in [1, 3-5]. The right part of this figure, section A-A', shows an in-plane view of a crack in the  $90^\circ$  ply. Since there are no well-defined boundaries between plies, the cracks extend in the ply thickness direction from one adjacent layer of fibers to another. For example, the crack in the  $90^\circ$  ply of Figure 1, section AA', extends between the  $0^\circ$  fibers and the  $+45^\circ$  fibers. In each layer these cracks grow in the direction of fiber axis, hence the crack planes have different orientation in each layer. On the boundaries between layers the cracks intersect.



It should be emphasized that the type of damage shown in Figure 1 can exist only when the matrix cracks do not propagate into the fibers, except for infrequent splits and breaks. This is the case in 6061-Al matrix reinforced by boron or SiC fibers, providing that the matrix is overaged, as-fabricated, or in T4 temper. Results obtained on composites with hardened (T6) matrices suggest that matrix cracks may break many fibers and as a result, the fatigue strength of these composites is very low.

The damage process shown in Figure 1 was modelled on the basis of the following premises:

- a) Cracks grow in a ply when the matrix experiences cyclic plastic straining. Monotonic plastic straining does not cause damage. When the undamaged composite is loaded within its shakedown limit, i.e., where the matrix is strained elastically, no damage takes place. This was indicated by experiments in [1].
- b) When, as a result of matrix cracking, the crack accommodation strain replaces the cyclic plastic strain, the matrix or ply ligaments between the cracks return to elastic state. If this happens in all plies, the cracked composite reaches a shakedown state. Damage accumulation stops. A saturation damage state is observed in experiments.

Figure 2 shows an example of saturation damage states reached at different levels of maximum cyclic stress  $S_{max}$ , for  $R = S_{min}/S_{max} = 0.1$ , in B-Al plate specimens. The change in the elastic modulus of the specimen, measured at unloading from  $S_{max}$ , was used as an overall measure of damage in the plate. Figure 3 is a collection of experimental points, each representing a saturation damage state. It also shows a comparison between the experimentally determined "damage limit" and the calculated shakedown limit of the composite plate specimen.

On the basis of these premises, we posed the problem to find crack density in each plastically strained ply such that the composite laminate reaches an elastic state under the applied cyclic load. This elastic state

should be identical with the experimentally observed saturation damage state at the same load cycle.

The model of this damage process was developed from several components, and in several steps. First, the actual loading problem was replaced by an equivalent one. In an experiment each specimen is loaded in a certain constant cyclic range,  $S_{\max}-S_{\min}$ . Initially, the composite deforms plastically, cracks start to grow after 50,000 cycles or so, then cracking and plastic straining coexist until the saturation state is reached after 500,000 or more cycles. The equivalent problem was formulated in such a way that the composite was first cycled to a steady state between  $S_{\max}$  and  $S_{\min}$ . Then, the loading range was reduced to an elastic range containing  $S_{\min}$ . Next, an increment of load was applied simultaneously with an increment in crack density, and the composite was loaded by several plastic cycles to a new steady state. This incremental process was repeated until the entire load range,  $S_{\max}-S_{\min}$  was accommodated. The final damage states reached in both the actual and equivalent loading problems should be identical. This can be ascertained on the basis of fatigue experiments under variable load [1, 2] which show that the stiffness loss in the saturation damage state depends only on the final stress range, and that the same stiffness loss can be reached either at constant range, or when the range is expanded incrementally, as in the equivalent problem.

To analyse the equivalent problem, we utilized both plasticity and damage analysis of laminated composite plates. The plasticity theory was developed earlier by Dvorak and Bahei-El-Din [6, 7], in stress space, and by Wung and Dvorak [8] for strain space applications which arose in the present problem. The self-consistent damage theory of cracked plies [9, 10], as well as a new theory which takes into consideration cyclic plastic straining at cracks were

utilized in damage analysis.

Results are shown in Figures 4 to 12. For simplicity, we use the (0/90)<sub>s</sub> layup in this illustration. Figure 4 shows the initial relaxation surfaces of the plies of the crossply laminate in the overall strain space where the  $\epsilon_{33}$  is the laminate strain in 0° direction. Each relaxation surface is described by an equation of the type [8]

$$\bar{y} = 2G_m^2 (\bar{\epsilon} - \bar{\beta})^T A_{me}^T C A_{me} (\bar{\epsilon} - \bar{\beta}) - \kappa^2 = 0 \quad (1)$$

where  $G_m$  is the matrix shear modulus,  $\bar{\epsilon}$  is the overall strain vector (6x1),  $\bar{\beta}$  is the position of the current center of the surface in  $\bar{\epsilon}$  space,  $A_{me}$  is the elastic matrix strain concentration factor (6x6), which relates overall uniform strain  $\bar{\epsilon}$  and average of the elastic matrix strain  $\epsilon_{me}$  in a given ply;

$$\epsilon_{me} = A_{me} \bar{\epsilon}, \quad (2)$$

and  $C$  is a (6x6) constant matrix;  $\kappa$  is the matrix yield stress in simple shear.

Now, when cracks are added, the concentration factors in the ligaments between cracks are reduced, because a part of the overall strain is now taken up by cracks. As a result, the  $A_{me}$  for the matrix within these ligaments are also reduced, and the surface (1) expands in the  $\bar{\epsilon}$  space. Figure 5 shows an example for the 90° ply of our crossply laminate. The expansion of the surface may take place only when the cracks are open, i.e., it affects only the tension branch of the surface in the  $\epsilon_{33}$  direction which coincides with the crack opening strain. The crack density is measured by a constant  $\beta$  defined as  $\beta = 2an$ , where  $2a$  is ply thickness (in mm), and  $n$  is the number of cracks in the ply per 1 mm of ply length. At  $\beta = 1$ , the average distance between the cracks is equal to  $2a$ . Similarly expanded relaxation surfaces can

be found for the  $0^\circ$  ply.

When the laminate is loaded into the plastic region, the relaxation surfaces are translated to contain the prescribed loading path. Figures 6-8 show these translations for undamaged laminate. Figure 8 indicates that steady state response of the laminate can be reached after the second cycle of loading. In general this is observed in both damaged and undamaged laminates after few loading cycles.

In the modelling procedure, the actual loading - constant cyclic load - was replaced by incremental cyclic loading which eventually attains the amplitude applied in experiments. The composite laminate was first loaded to the maximum stress of the actual cycle, then to the minimum stress, and the incremental cyclic loading sequence was applied so that the minimum stress was always included in the incremental cycle.

In the plasticity part of the damage analysis, the composite was first brought to a steady state that is actually reached in experiments after several cycles. Initially, this was a shakedown state in an undamaged composite. Next, an increment of load was applied, and at the same time an increment in crack density was allowed in each ply that would be loaded plastically during the new load cycle. A new shakedown state was found for this level of damage. This incremental procedure was repeated until the entire prescribed load cycle was absorbed. In the final saturation state the composite is in a shakedown state, the plastic strains which were originally applied to the material are accommodated by opening and closing of cracks.

During the incremental loading process, the relaxation surfaces expand to accommodate the loading path, and they also translate in such a way that the path always coincides with the largest "diameter" of each surface. This is a consequence of cyclic plastic straining to a steady state, which is performed

between crack increments. In other words, the composite experiences maximum possible amount of cyclic elastic straining and it also contains a minimum amount of cracks which are necessary to reach a given saturation damage state. Figures 9 and 10 show these expanded surfaces at two levels of  $S_{max}$ .

This analysis was performed for three different B-A<sup>2</sup> laminate layups. Figure 11 indicates the results obtained for  $0_8$  and  $(0/90)_{2s}$  laminates. The loss of unloading elastic modulus in saturation damage state is plotted as function of applied stress range. Experimental results reported in [1, 2] are shown for comparison. Figure 12 shows similar results for the laminate discussed in connection with Figures 2 and 3. The experimental data in Fig. 12 were taken from Figure 3.

Finally, Figure 13 indicates the changes in  $0^\circ$  fiber stresses as a function of damage, which is represented by the magnitude of reduction of elastic modulus in saturation damage state. The calculated maximum stress reached in the  $0^\circ$  fibers is also shown in the figure for each laminate. We note that the  $0_8$  and  $(0/90)_{2s}$  plates were from one batch of material, which was apparently weaker, while the 15-layer plate was from another batch of stronger material.

The conclusion one can obtain from these data and theoretical predictions is that the laminate endurance limit is reached when the fiber stresses in the  $0^\circ$  ply reach a value which is equal to cyclic fiber strength. Damage has a strong effect both on stiffness and strength of the laminates. These results will appear in complete form in the forthcoming Ph.D. dissertation by Mr. C.J. Wung.

## 2.2 Mechanics of First Ply Failure

In this study, the mechanics of crack initiation in an elastic fibrous ply was explored. Cracks were assumed to initiate from a nucleus created by

localized fiber debonding and matrix cracking. Conditions for onset of unstable cracking from such nuclei were evaluated with regard to interaction of cracks with adjacent plies of different elastic properties. It was found that cracks may propagate in two directions on planes which are parallel to the fiber axis and perpendicular to the midplane of the ply. It was found that crack propagation in the direction of the fiber axis controls the strength of thin plies, while cracking in the direction perpendicular to the fiber axis determines the strength of thick plies. The theory relates ply thickness, crack geometry, and ply toughness to ply strength. It predicts a significant increase in strength with decreasing ply thickness in constrained thin plies. The strength of thick plies was found to be constant, but it could be reduced by preexisting damage. Strength of plies of intermediate thickness, and of unconstrained thick plies was evaluated as well.

While the theory is applicable at any crack density, it was compared with experiments performed at low crack densities. Figures 14 and 15 show comparisons of the theory with experimental data obtained for first ply failure in 90° plies of different thicknesses. The theory predicts two different strength curves. First, for thin plies one obtains the steep curves shown in the two figures. The equation of this curve is

$$(\bar{\sigma}_{22})_{cr} = [4G_{Ic}(L)/\pi \xi_I \Lambda_{22}^0 a]^{\frac{1}{2}} \quad (3)$$

where  $(\bar{\sigma}_{22})_{cr}$  is the transverse ply strength,  $G_{Ic}(L)$  is ply toughness for the case of crack propagation in the fiber direction,  $\xi_I < 1$  is a coefficient which describes the reduction of crack energy caused by interaction of the crack with an adjacent ply,  $\Lambda_{22}^0$  is the relevant component of the crack tensor obtained in [10] and  $2a$  is ply thickness. Second, for thick plies one obtains

the prediction that the fracture stress is constant, and given by

$$(\bar{\sigma}_{22})_{cr} = [2G_{IC}(T)/\pi \Lambda_{22}^0 \delta_{IC}(T)]^{1/2} \quad (4)$$

where  $G_{IC}(T)$  is the ply toughness for a crack propagating in the direction perpendicular to the fiber axis, and  $\delta_{IC}(T)$  is the critical width of the initial flaw.

In practice,  $G_{IC}(T)$  and  $\delta_{IC}(T)$  may not be known, and the strength of thick plies must be measured experimentally. However, (4) predicts that it is constant. If initial damage is present such that  $2\delta_{IC}(T) = 2a$ , then the strength of thick plies is reduced to that given by (3), at large values of  $a$ .

A complete description of these results is given in [11, 12].

### 2.3 Distributed Damage in a Ply

In our work on distributed damage in a ply, attention was focused on the loss of stiffness of the laminate and on progressive cracking of the transverse ply. As far as loss of stiffness is concerned, our aim has been to analyze the problem for transverse cracks and to treat the problem without the introduction of additional material parameters (as for example in the internal variable approach). In particular we have developed the self-consistent model [9-11, 13-15] and this technique appears to be adequate in practice.

We have also compared the self-consistent model with the work of Delameter, Herrmann and Barnett [16, 17]. These latter authors addressed some problems associated with a periodic two-dimensional distribution of aligned cracks in an infinite solid. The geometry assumed by Delameter, Herrmann and Barnett has long-range order. On the other hand the self-consistent model assumes a random distribution of cracks. Nevertheless it is possible to compare the two approaches. It is significant that there is good agreement.

In addition we have used shear lag theory to estimate the loss of stiffness of cracked laminates. However it must be emphasized that the original version of shear lag as it applies to cross-ply laminates [18, 19] contains numerous mistakes. In fact, some effort was required to provide a simple clear basis for shear lag theory. Once the required framework had been established, it was possible to do the necessary stress analysis. Of course, it is well known that shear lag theory involves one additional constant. But it is possible to give an easy way of finding the shear lag parameter from existing data. With this information one can easily calculate the shear lag prediction of the loss of stiffness. It is quite remarkable that all these methodologies are in good agreement amongst themselves and with experiment. Thus from a practical point of view either the self-consistent or shear lag models are to be preferred because of their simplicity.

A further bonus in our analysis of the various models emerged when we compared shear lag theory with some finite element calculations. Indeed our analysis indicates that shear lag gives quite acceptable results for the stresses (and strains) except in the immediate vicinity of the crack surfaces. And this explains why shear lag gives a reasonable prediction for loss of stiffness.

A significant achievement of the last year has been the formulation of a good analytical model for progressive cracking in cross ply composite laminates. In the first place the work of the two P.I's - described elsewhere - provides a simple approach to first ply failure. At the time of writing, this approach has not been extended to handle progressive cracking. Accordingly an alternative simple model for progressive cracking was sought [20]. It turns out that shear lag theory which must be the simplest possible model coupled with statistical fracture mechanics gives some particularly useful results. It is perhaps important to emphasize that the proposed progressive cracking model



has little in common with the earlier models of Bailey and his co-workers [18, 19].

The work on damage propagation across and along ply interfaces has been extensive. The most comprehensive stress analysis of this sort of problem has been undertaken by Erdogan and a succession of students [21, 22 and the references contained therein]. As far as this project is concerned a major difficulty is due to the fact that the specific geometries which are encountered in transverse ply cracking have not been reported in the literature. Further, it is not possible to obtain from the published literature any comprehensive results which are particularly relevant to cross-ply graphite-epoxy laminates. In order to provide the required computer algorithms, and thus an extensive data base, a succession of problems of gradually increasing complexity has been analyzed. For some model problems the results obtained have been in complete agreement with published work. But for other, more difficult, problems we have been unable to reproduce some of the published results. At this time it is not possible to identify the difficulties with precision and thus arrive at definitive answers. However, it can be mentioned that we have attempted several problems using the analytical techniques developed by Erdogan and his students [21, 22]. So far our attempts have been unsuccessful. It is fair to summarize our current position by noting that we are unable to get convergence of the recommended algorithms. The source of the problem is not clear at the time of writing.

In order to get some additional insight into these problems, we obtained some finite element solutions of the model problems. The solutions were obtained by a straightforward application of the standard ABAQUS code. This choice was determined by the fact that we needed to use a well understood, reliable program.

Obviously this problem area is under intense study. Future reports can be expected to contain precise scientific assessments of the various issues.

By way of contrast, it is pleasing to be able to report good progress in the study of crack growth across the ply interface into an adjacent ply. This process often causes fiber breaks in the next ply and contributes significantly to ply damage and to stiffness loss. To solve this problem, it was necessary first to calculate interaction energies and crack opening displacements for cracks perpendicular to fibers: Laws [23]. These results have been incorporated into stiffness change calculations [24]. In addition to the determination of self-consistent models, we have also investigated the appropriate Hashin-Shtrikman bounds [24]. As ever, the self-consistent model lies within the bounds. Also some results for energy releases rates have been obtained.

We are also examining the possibility of modeling the crack extension across an interface with the help of discrete Dugdale-type zones.

Finally, we discuss the work in progress on the effect of damage on the behavior of laminated plates. Here we are concentrating on the modeling of extensively damaged regions. The formulation of constitutive equations for plies containing extensive matrix damage has been completed. Also fiber damage resulting in cracks perpendicular to the fiber has been analyzed. In addition, we have done some assessment of the propagation of damage in a laminate. Initial damage zones (due to impact or projectiles) extending through the thickness of the laminate were considered. On the basis of these preliminary studies, it is clear that problems associated with the spread of damage from extensively damaged regions are not easily solved. This is a major problem area in our continuing work.

### 3. SIGNIFICANT ACHIEVEMENTS

Important technical results have been obtained in analysis of damage both in metal and polymer matrix composites. The results reported in Section 2.1 above highlight the achievements in analysis of fatigue damage in fibrous metal matrix systems. In this case the damage process is quite complex, not only in terms of crack geometry, but also because of the interaction of plastic deformation and cracking during cyclic loading. Modelling of the process was made possible by formulation of a different, equivalent problem. However, the most significant factor in the success of the procedure was that we set out to model the terminal state of damage, i.e., the saturation damage state. No attempt was made to follow evolution of damage through the initial accumulation stage of 500,000 cycles or so, which is of limited practical significance but which involves complex deformation and cracking processes.

Damage processes in polymer matrix systems were analysed in several ways. First, the analysis of first ply failure clarified the conditions which govern nucleation and propagation of individual cracks in a polymer matrix fibrous ply. These conditions are valid not only for the first crack, but also for all subsequent cracks, providing that the local stresses acting on the new crack nucleus are properly evaluated. Therefore, we are now in the position to connect progressive cracking in a composite ply to material properties such as ply toughness, and ply elastic properties, to ply thickness, laminate geometry, and to initial flaw size.

Our efforts in analysis of progressive cracking of a fibrous ply have produced additional technical achievements. As a part of this effort, we compared various crack array models in terms of their predictions of stiffness reduction in a cracked ply. The self-consistent model [9, 10] was compared

with the two-dimensional periodic array results [16, 17] and also with rederived shear lag model results [20]. In these comparisons we considered the composite laminate geometry shown in Figure 16. The longitudinal and transverse Young's moduli for the unbroken plies are  $E_l$  and  $E_t$  respectively. Also  $E_0$  is the Young's modulus for the uncracked laminate (in the direction of the fibers of the  $0^\circ$  plies).

A useful test of the self-consistent model can be obtained by comparison with the work of Delameter, Herrmann and Barnett [16, 17]. These authors consider a two-dimensional periodic array of cracks in an isotropic solid and, amongst other things, calculate the associated loss of stiffness. Our self-consistent results given in [14] refer specifically to laminates with the following lay-ups:

$$(0_4/90)_s, (0_3/90_2)_s, (0_2/90_3)_s, (0/90_4)_s.$$

It is simple enough to extract the required reductions in stiffness from [17] and compare the results with the self-consistent model, see Table 1. Whilst the comparison is remarkably good, it is not decisive since Delameter, Herrmann and Barnett [17] only consider isotropic solids.

It is reassuring to have agreement between the self-consistent model and the Delameter, Herrmann, Barnett analysis, but it is surely more important to compare the respective models with experiment. A major difficulty is the sparsity of experimental evidence on the relationship between stiffness loss and crack density. However, the effectiveness of the self-consistent model in predicting the experimental results of Highsmith et al. [24] is shown in Figure 17. We note that Figure 17 also includes the prediction of shear lag theory. It is useful to record that shear lag theory gives the following formula for

the reduction in stiffness [20]

$$\frac{E}{E_0} = \left(1 + \frac{\beta E_t}{2\lambda b E_\ell} \tanh \frac{2d\lambda}{\beta}\right)^{-1}.$$

The parameter  $\lambda$  is the so-called shear lag parameter, but for practical purposes is best handled in the form  $(\lambda d)$ , which is dimensionless. For typical composites  $0.5 \leq \lambda d \leq 1.5$ . In fact for the  $(0, 90)_3^s$  E-glass epoxy laminates studied by Highsmith et al. [24],  $\lambda d = 0.8$ .

A significant achievement has been to use shear lag theory to predict progressive crack density as a function of applied loads [20]. Clearly such an analysis must take into account initial stresses. Accordingly the model has been formulated and solved. The process of transverse cracking can be followed in several ways. In the first place it is easy to show that the applied stress at first ply failure ( $\sigma_a^{fpf}$ ) may be found in terms of the critical energy release rate  $G_c$  and the other parameters from

$$\sigma_a^{fpf} = \left\{ \frac{\lambda b E_\ell E_0 G_c}{(b+d) E_t} \right\}^{1/2} - \frac{E_0}{E_t} \sigma_t^R,$$

where  $\sigma_t^R$  is the residual stress due to cool-down in the transverse ply. If we are given the usual data then the preceding formula provides a relationship between  $\sigma_a^{fpf}$ ,  $\lambda$  and  $G_c$ . Thus given  $\lambda$  and  $G_c$  we could predict  $\sigma_a^{fpf}$ . However, at the present time it is more appealing to regard  $\sigma_a^{fpf}$  and  $G_c$  as given, hence we can determine  $\lambda$  or, more usefully,  $(\lambda d)$ .

As for subsequent cracking, a major complication arises because the location of the next crack cannot be obtained by deterministic methods. Physically it is clear that, for a given cracked laminate, there is no way of determining the location of the next crack. Rather, we must proceed on a

statistical basis. Referring to Figure 18, suppose that in a laminate with transverse crack density  $\beta$ , existing neighboring cracks are at A and B. Then the next crack to occur between A and B will occur at some location C. Thus let  $p(y)$  be the appropriate probability density function for the site of the next crack: in other words the probability that the next crack will appear between locations  $y$  and  $(y + dy)$  is just  $p(y)dy$ . Thus if  $\sigma_a(y)$  is the applied stress to cause the next crack to appear at location  $y$ , the expected value of the applied stress to cause a laminate with crack density  $\beta$  to undergo additional cracking is

$$E(\sigma_a) = \int_0^{2h} p(y) \sigma_a(y) dy ,$$

where

$$\begin{aligned} \sigma_a(y) = & (\sigma_a^{fpf} + \frac{E_0}{E_t} \sigma_t^R) \left\{ \tanh \frac{\lambda y}{2} + \tanh \frac{\lambda}{2} \left( \frac{2d}{\beta} - y \right) - \tanh \frac{\lambda d}{\beta} \right\}^{-1/2} \\ & - \frac{E_0}{E_t} \sigma_t^R \end{aligned}$$

The choice of probability density function is crucial and unknown. One extreme would be to assume the next crack to occur at the mid-point of the segment, thus giving

$$p(y) = \delta(y - h) ,$$

where  $\delta(y)$  is the Dirac delta function. The other extreme would be to assume that all locations in the ligament are equally likely, giving

$$p(y) = \frac{1}{2h} .$$

But a more appealing hypothesis, based on simple fracture mechanics, would be to assume that  $p(y)$  is proportional to the stress in the transverse ply:

$$p(y) \propto \sigma_t(y) \quad .$$

For the  $(0/90_3)_s$  E-glass reinforced plastic laminates studied by Highsmith et al. [24], the data demands that

$$\lambda d = 0.8 \quad .$$

Comparison between theory and experiment is shown in Figure 19.

In addition we have used some of the extensive data compiled by Wang et al. reported in [25] for T300/934 graphite epoxy laminates results are shown in Figures 20 and 21.

Bearing in mind that the above-mentioned shear lag theory must be the simplest possible theory of transverse cracking, it is remarkable that the use of a statistical fracture mechanics approach is as accurate as indicated in Figures 19, 20 and 21.

Turning now to the reduction in stiffness due to fiber breaks, it is inappropriate to give here any details of the analysis. But in order to give any information it is necessary to identify the appropriate crack density  $\alpha$ . Thus let  $N$  be the number of fiber breaks per unit volume and let  $a$  be the radius of the resulting penny-shaped crack at the broken end, then

$$\alpha = 8Na^3 \quad .$$

In order to determine the loss in stiffness due to fiber breaks, it was first essential to calculate the interaction energy for a penny-shaped crack in a transversely isotropic material. This was done in [23]. It was then reasonably straightforward to obtain the self-consistent model and the corresponding

Hashin-Shtrikman bounds etc. The loss in Young's modulus ( $E_L$ ) parallel to the fibers (and perpendicular to the cracks) is shown in Figure 22. Likewise the reduction in the longitudinal shear modulus is shown in Figure 23. We have computed all the necessary physical parameters, including energy release rates. The details are too involved to be reported here, but see Laws and Dvorak (26).

At the present time we have a range of results for stress intensity factors which have been obtained using ABAQUS. A sample is shown in Figure 24 where the normalized stress intensity factor for a single crack in a  $(0/90)_s$  graphite-epoxy laminate is plotted as a function of (non-dimensional) crack length. It is not possible to compare this solution with published work since none of the published work refers to the problem in hand. To be more precise, published work refers to laminates with constrained upper and lower surfaces whereas the real problem which is needed here concerns free upper and lower surfaces.

Clearly this subject is under intensive study. A complete analysis will be forthcoming in future reports.



#### 4. ACKNOWLEDGEMENT

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## 5. LIST OF PUBLICATIONS

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9. Laws, N. and Dvorak, G.J., "Shear Lag Theory for Cracked Composite Laminates," forthcoming.

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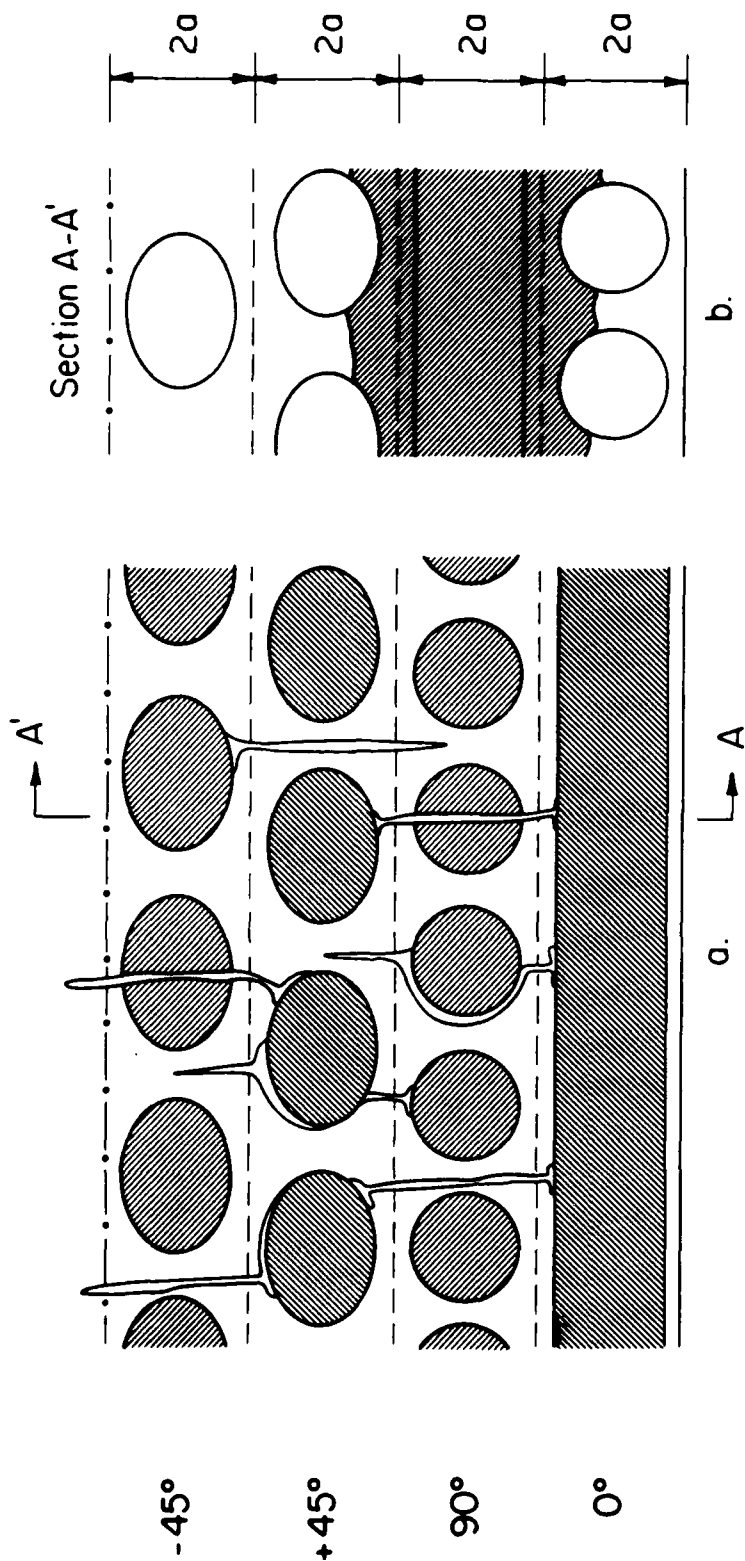
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	$\beta = 0.1$	$\beta = 0.3$	$\beta = 0.5$
	SCM	SCM	SCM
	DHB	DHB	DHB
$(0_4, 90)_s$	98 97	93 94	89 90
$(0_3, 90_2)_s$	95 94	85 85	77 78
$(0_2, 90_3)_s$	92 91	77 77	65 64
$(0, 90_4)_s$	90 85	70 66	53 54

Table 1: The predicted remaining % of the initial stiffness at the indicated crack densities according to the Delameter, Herrmann, Barnett [9] analysis and the self-consistent model.



Schematic reconstruction of various observed crack patterns in B-Al<sub>2</sub> laminates

- Longitudinal section of a  $(0/90/\pm 45)_s$  plane
- In-plane view of a transverse crack in the  $90^\circ$  layer of the  $(0/90/\pm 45)_s$  plate

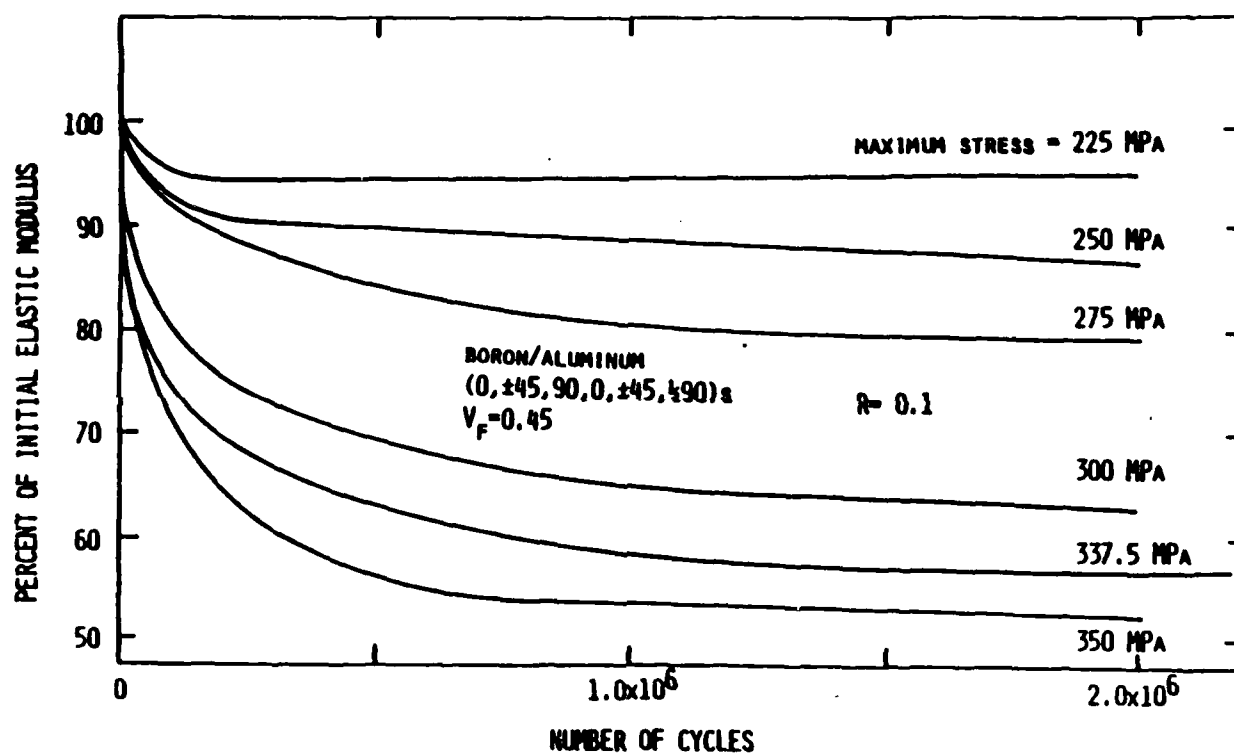


Fig. 2 Change in elastic modulus of B-Al specimens at different values of  $S_{max}$ , [1]



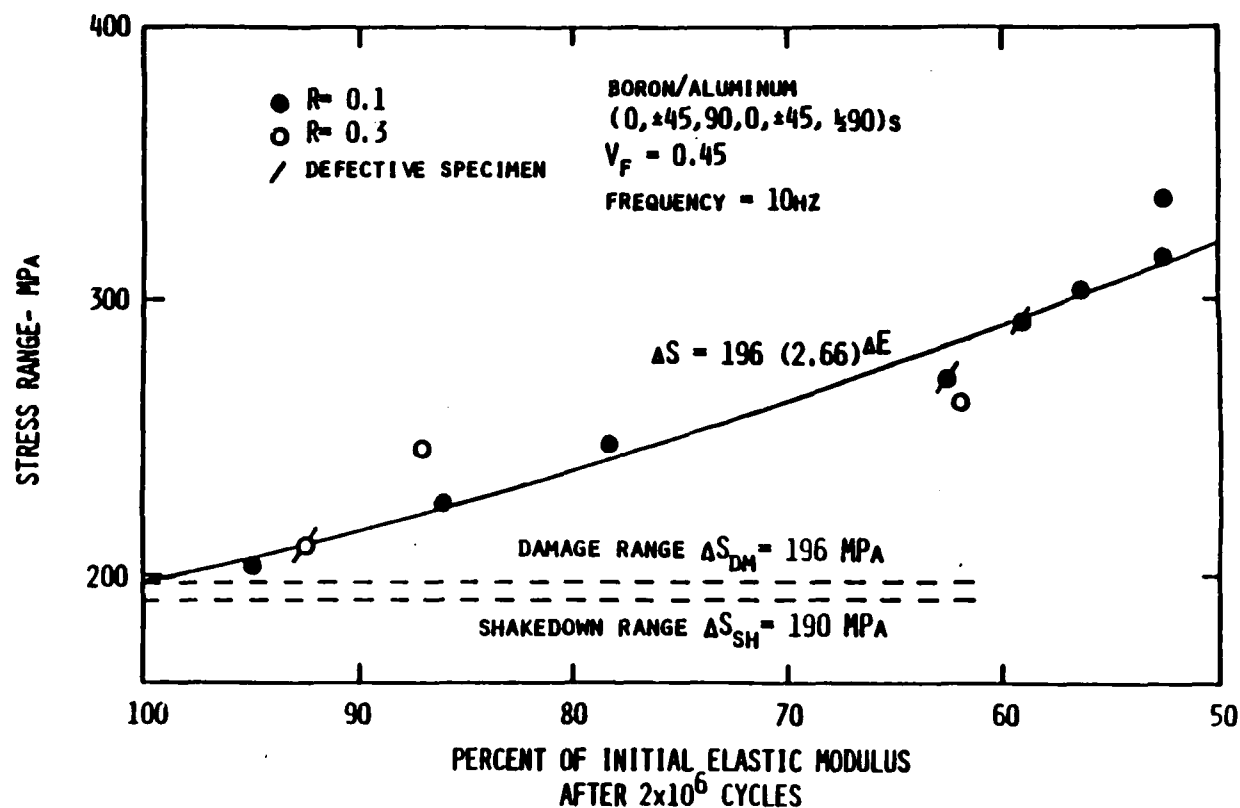


Fig. 3 Change in elastic moduls of B-Al specimens related to applied stress range  $\Delta S$ , [1]

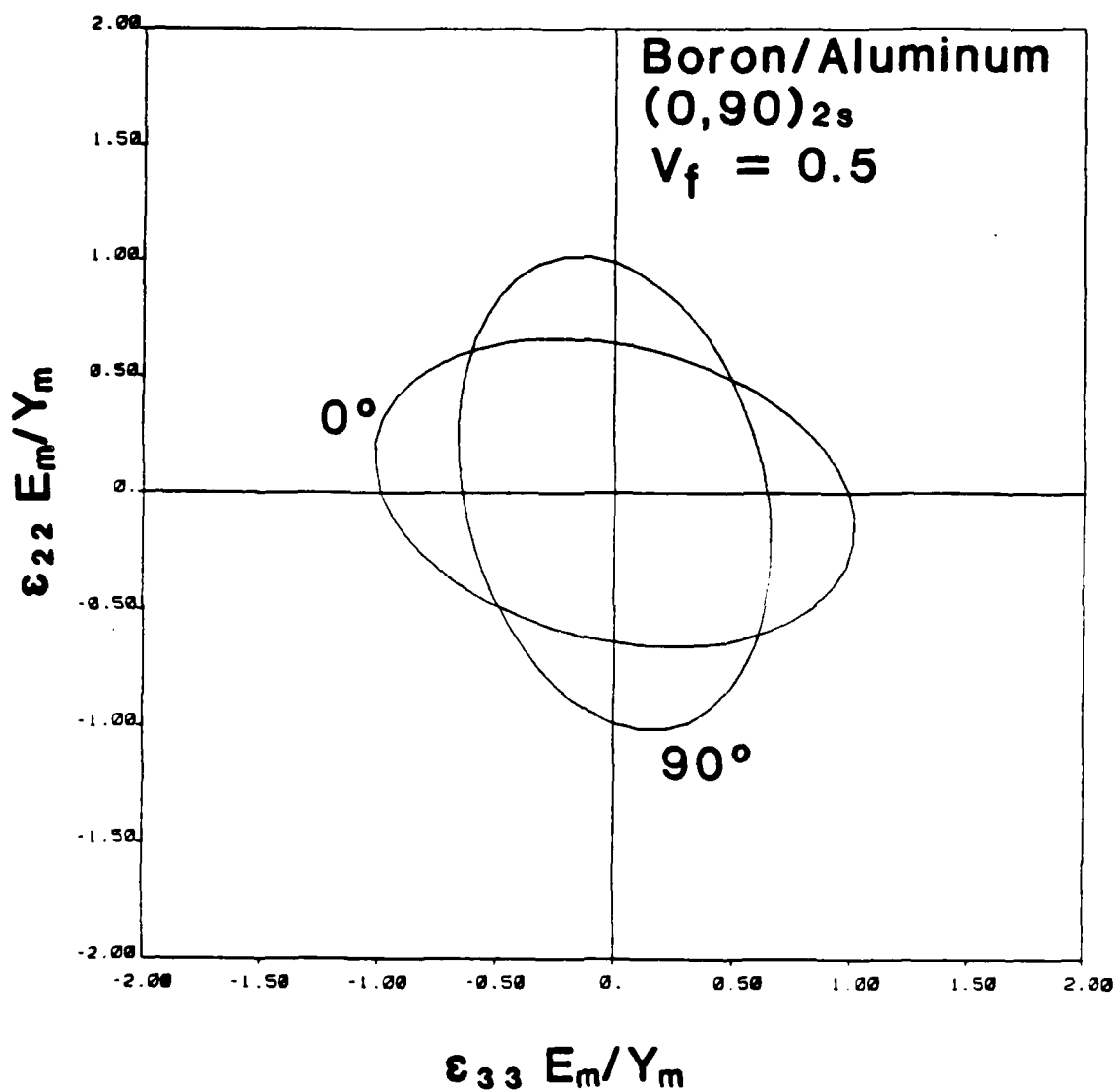


Fig. 4 Initial relaxation surface of a B-Al<sub>2</sub> laminated plate loaded by in-plane biaxial normal strains

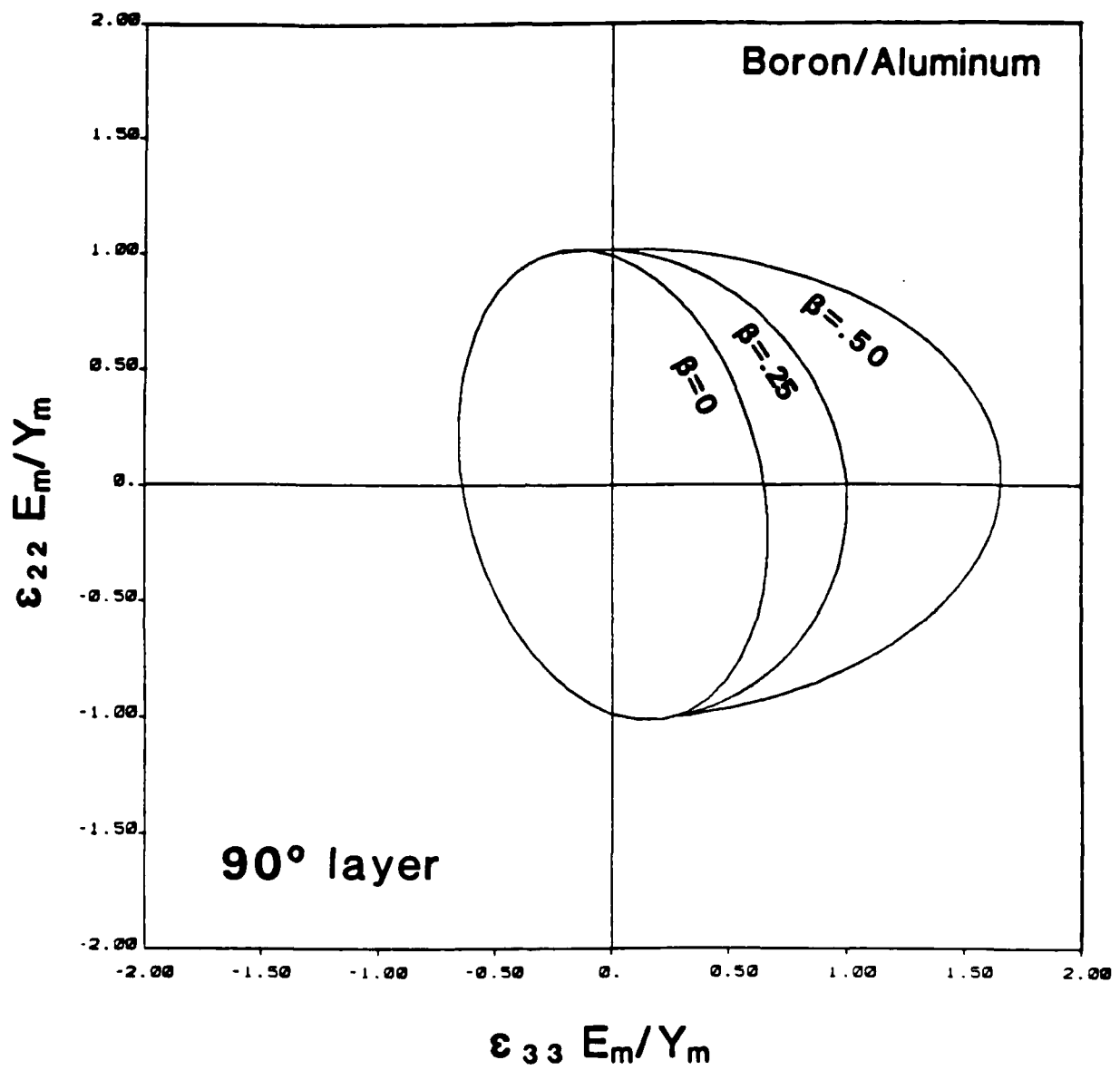


Fig. 5 Relaxation surfaces of the 90° layer at different crack densities

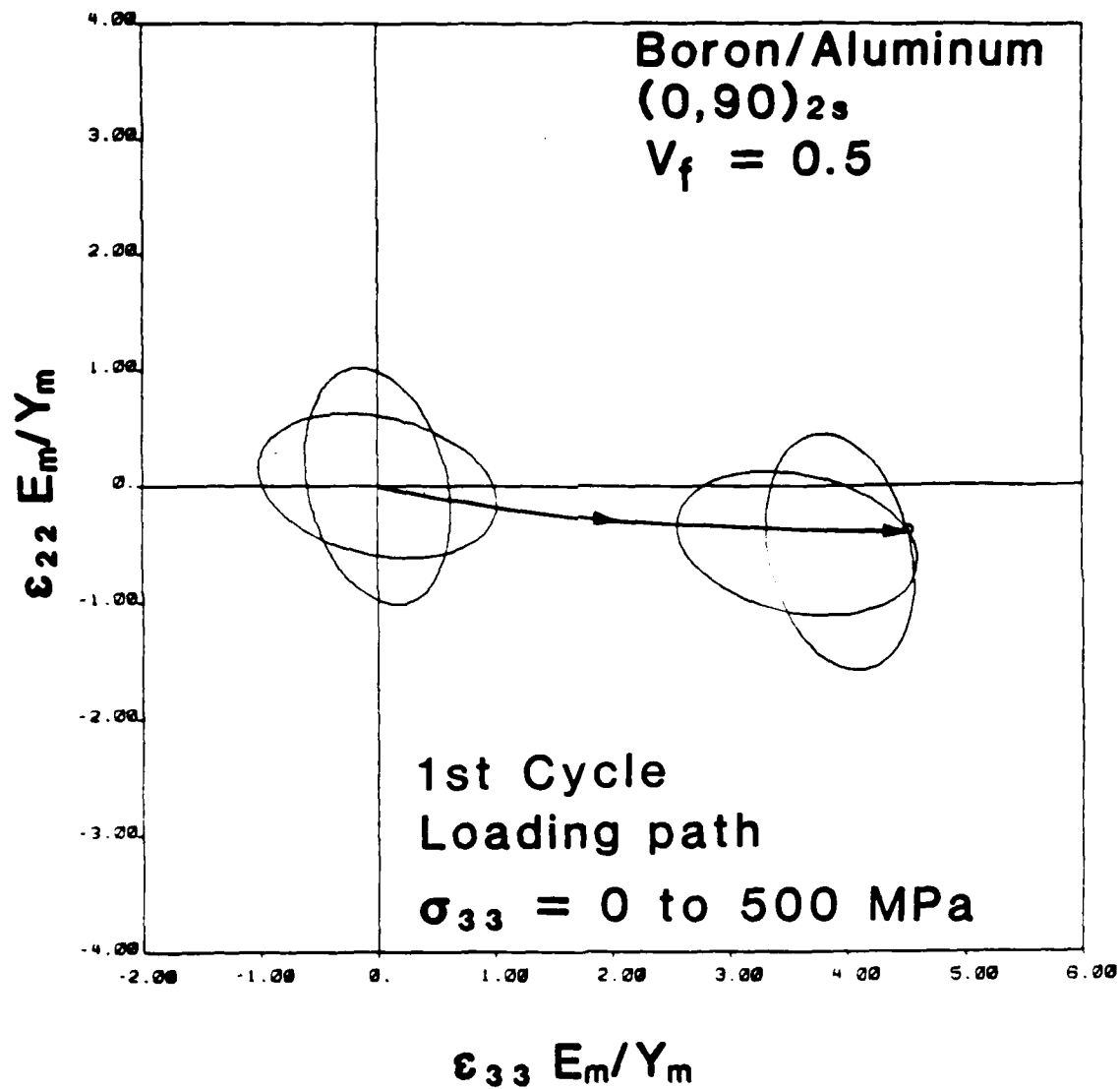


Fig. 6 Motion of the laminate relaxation surface during the first cycle

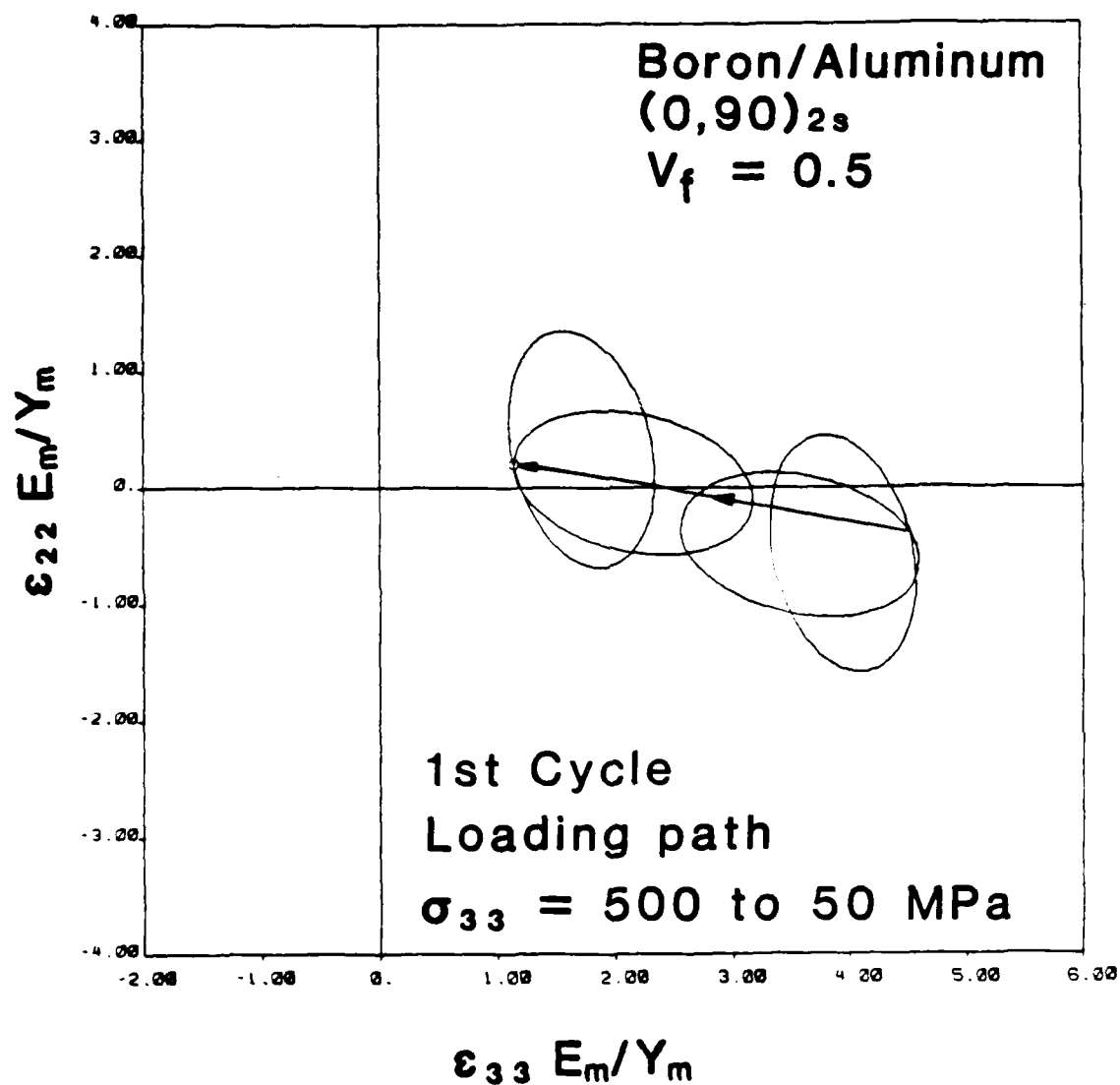


Fig. 7 Motion of the laminate relaxation surface during the first cycle

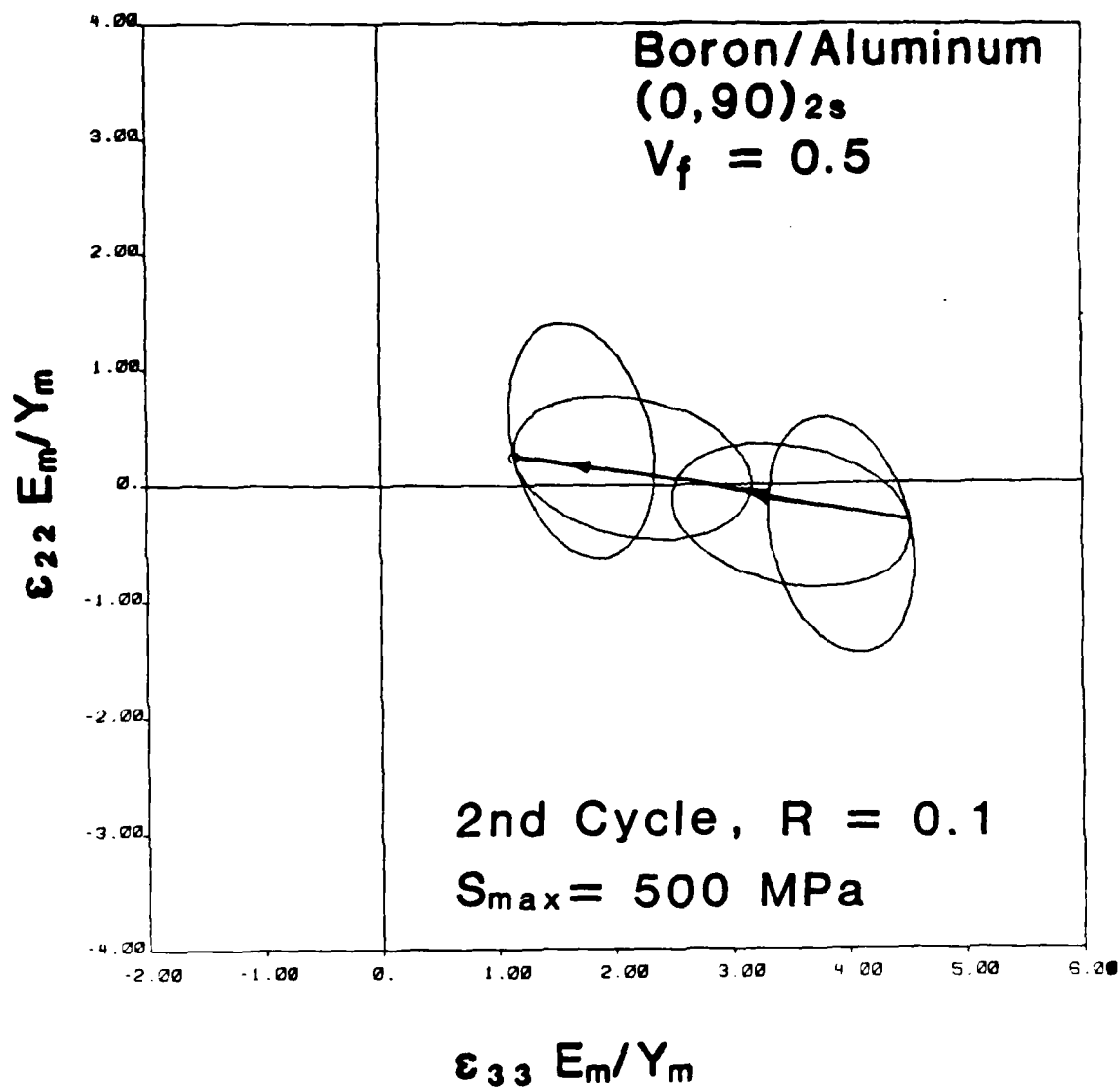


Fig. 8 Motion of the laminate relaxation surface during the second cycle

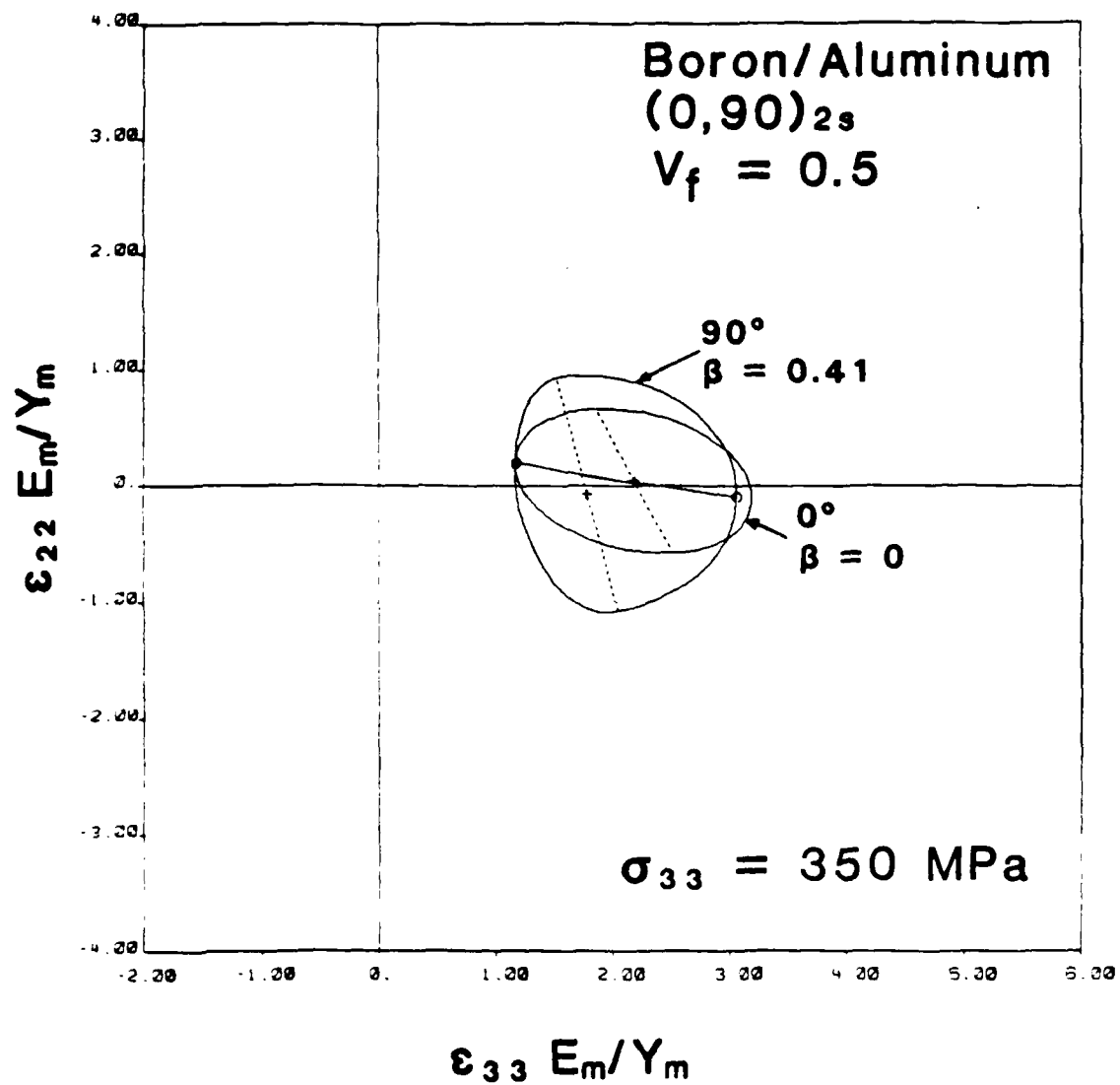


Fig. 9 Expanded 0/90 relaxation surfaces at  $S_{\max} = 350 \text{ MPa}$

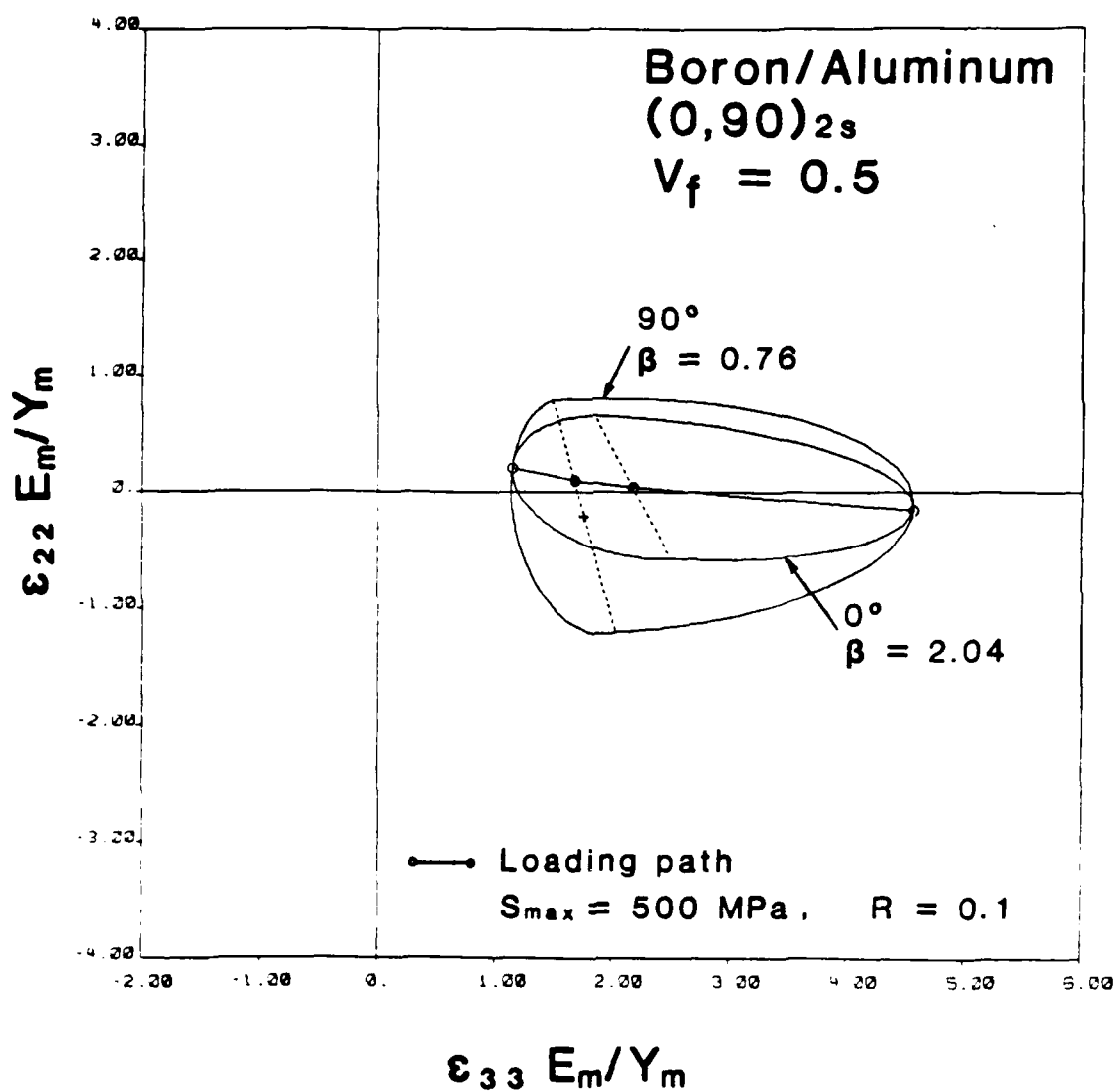


Fig. 10 Expanded 0/90 relaxation surfaces at  $S_{max} = 500 \text{ MPa}$



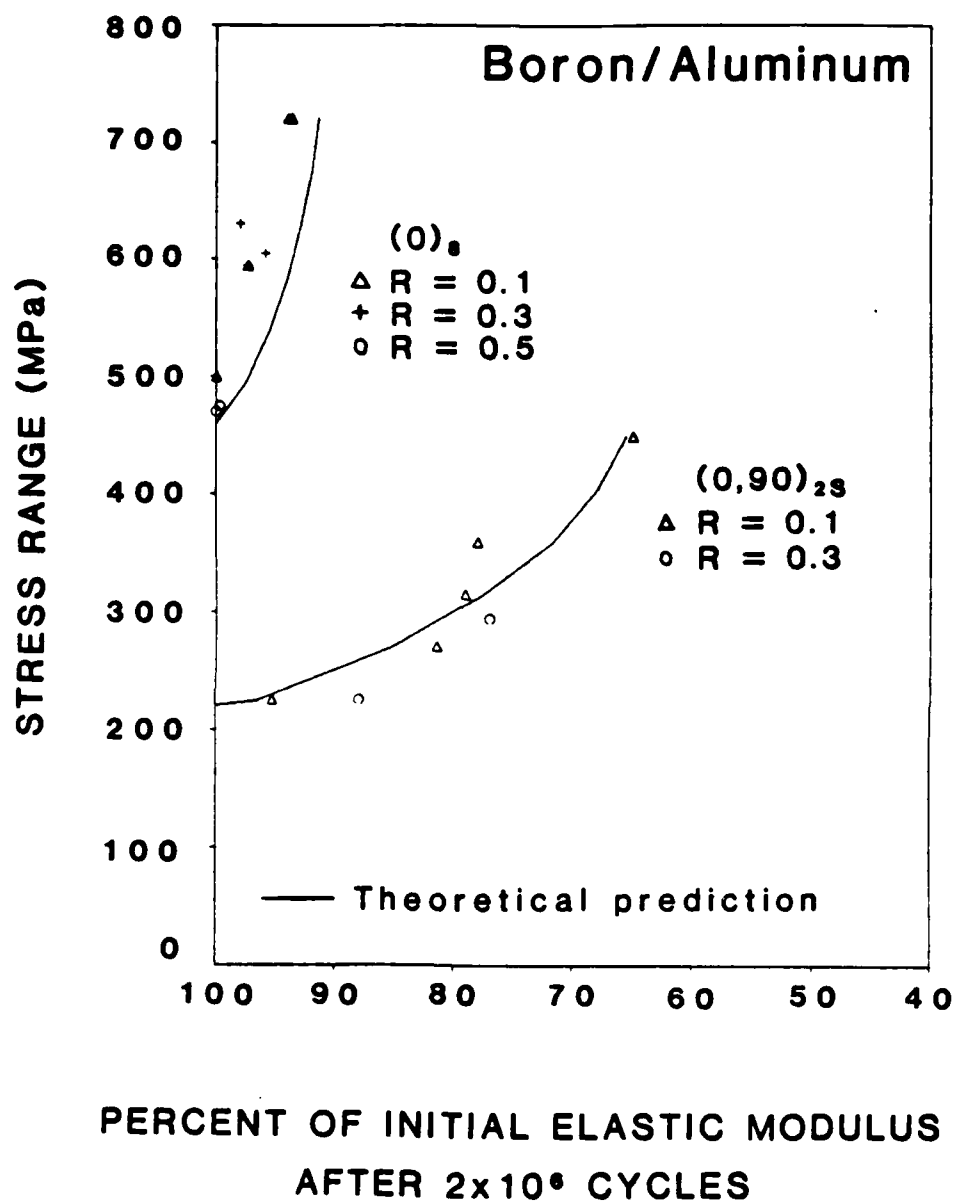


Fig. 11 Change in elastic modulus of B-Al plates related to applied stress range; comparison of theory and experiment

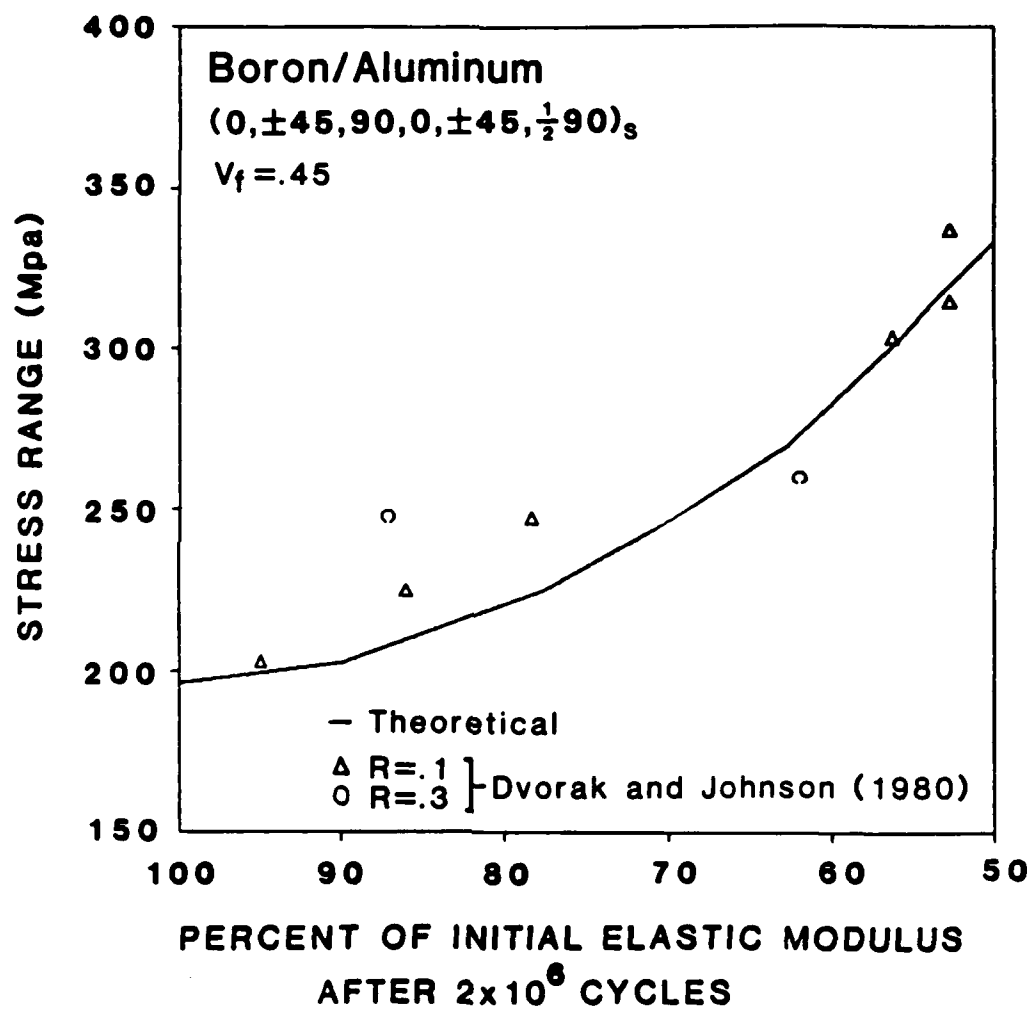


Fig. 12 Comparison of predicted laminate stiffness change after  $2 \times 10^6$  cycles with experimental data

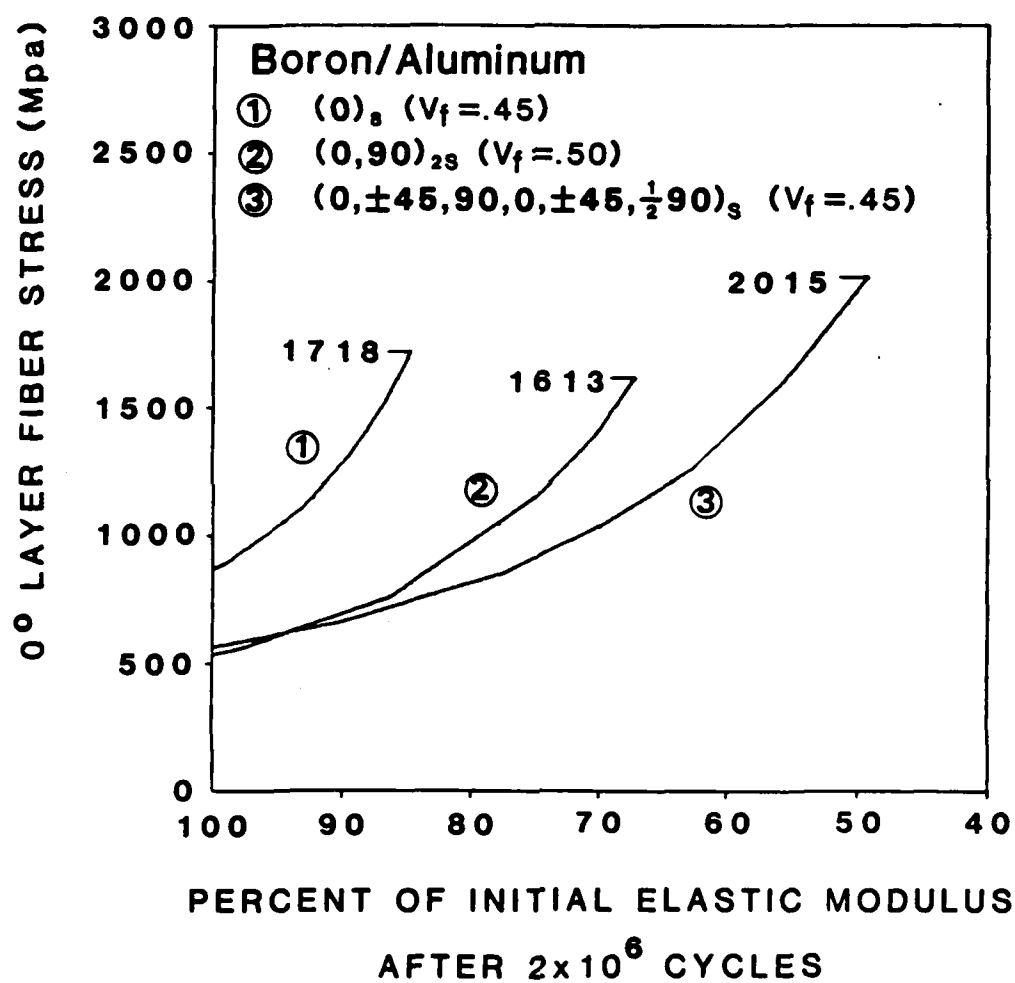


Fig. 13 Change in axial layer fiber stresses with stiffness loss in saturation damage state

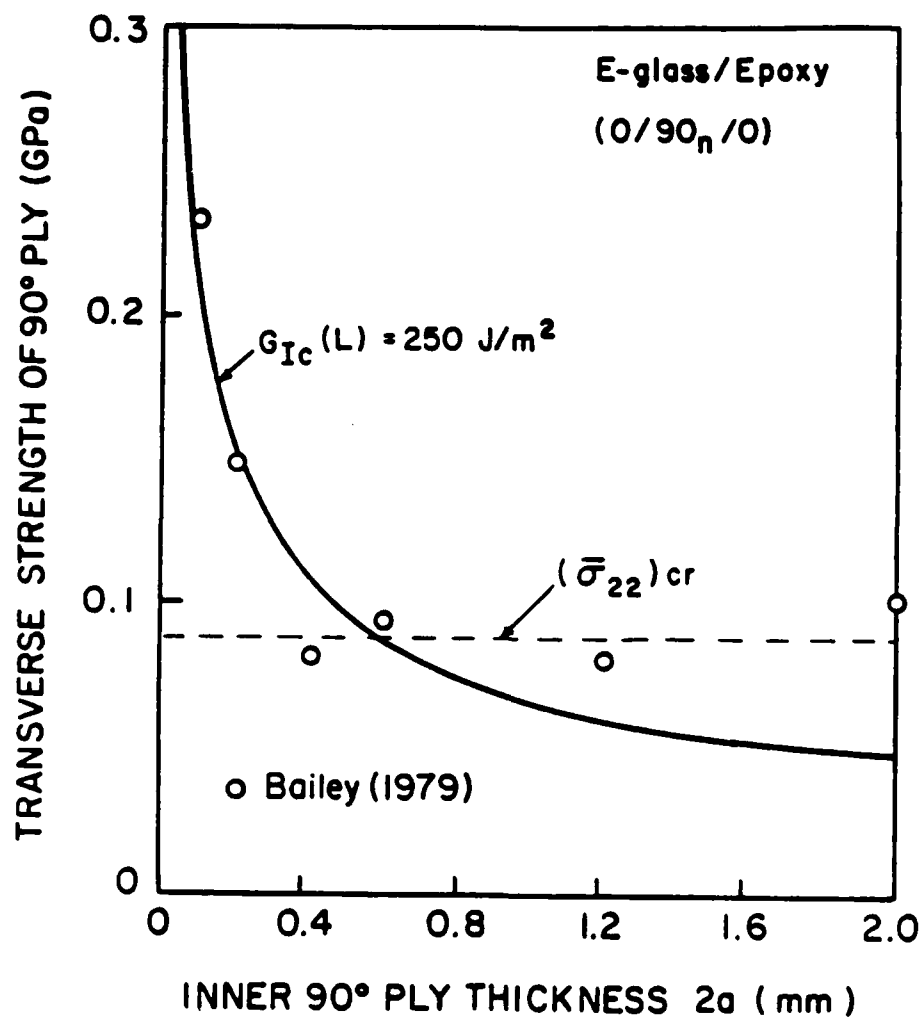


Fig. 14 Comparison of theoretical results with experimental data for stresses at first ply failure in a  $[0/90_3]_s$  E glass-epoxy laminate

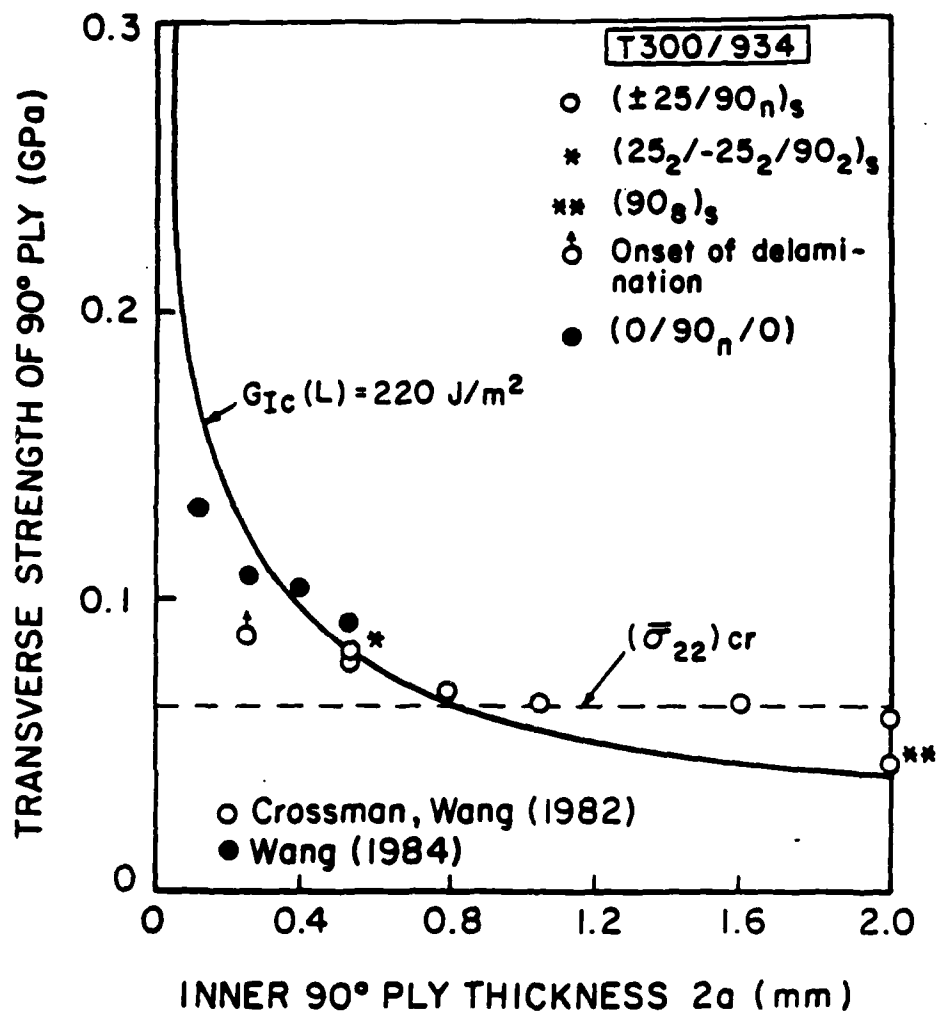


Fig. 15 Predictions and measurements of stiffness as a function of crack density for a graphite-epoxy laminate

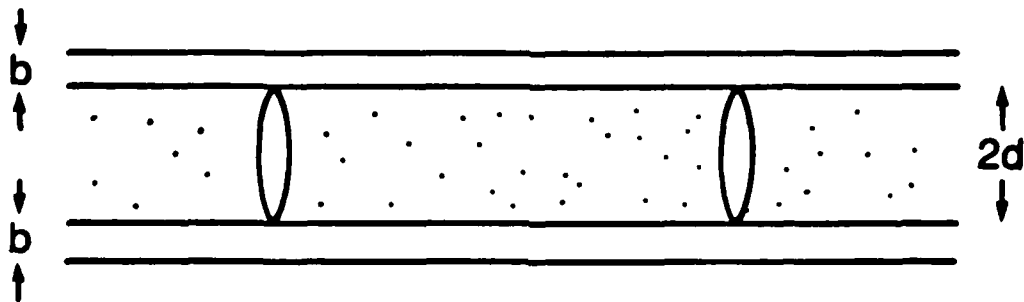


Fig. 16 Cross ply configuration. Outer ply thickness =  $b$ , inner ply thickness =  $2d$ . Average distance between cracks =  $2d/b$ .

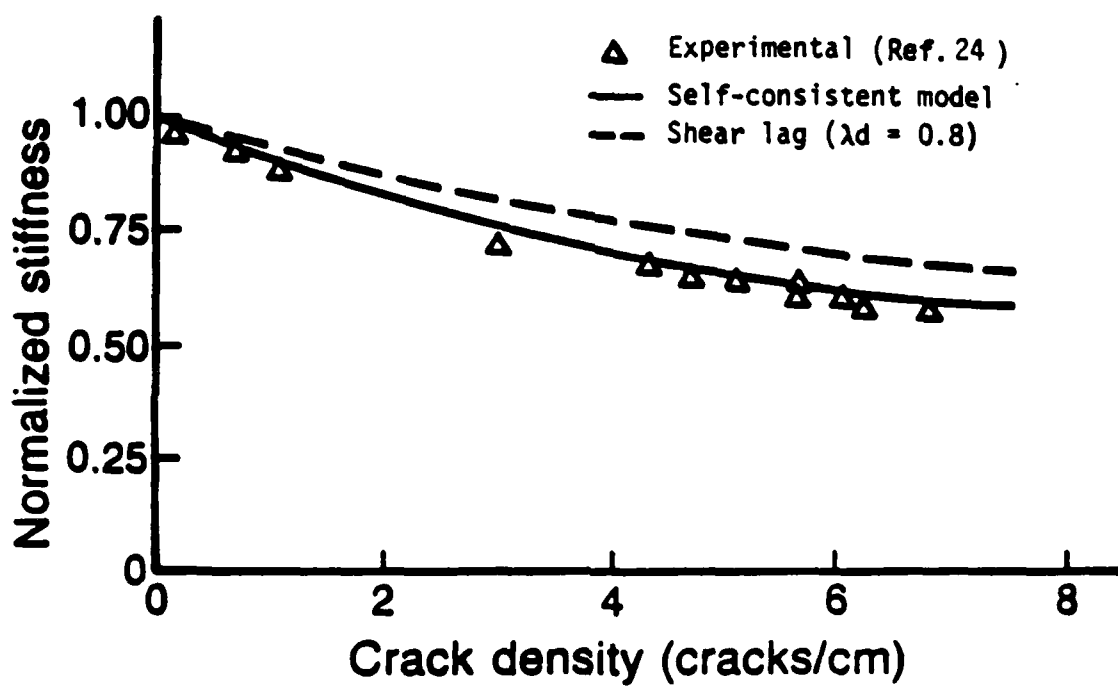


Figure 17. Predictions and measurements of stiffness as a function of Crack density for a  $[0/90_3]_s$  E glass-epoxy laminate.

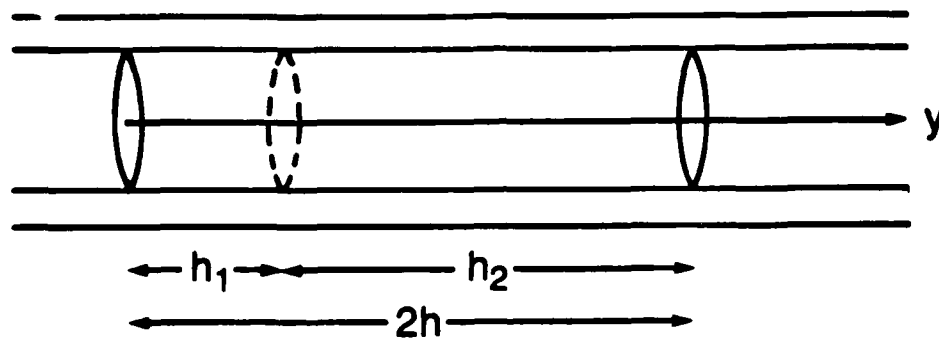


Fig. 18 For given cracks at  $y = 0, 2h$ , the figure shows the potential location of the next crack in the ligament



# GFRP (0, 90<sub>3</sub>)s

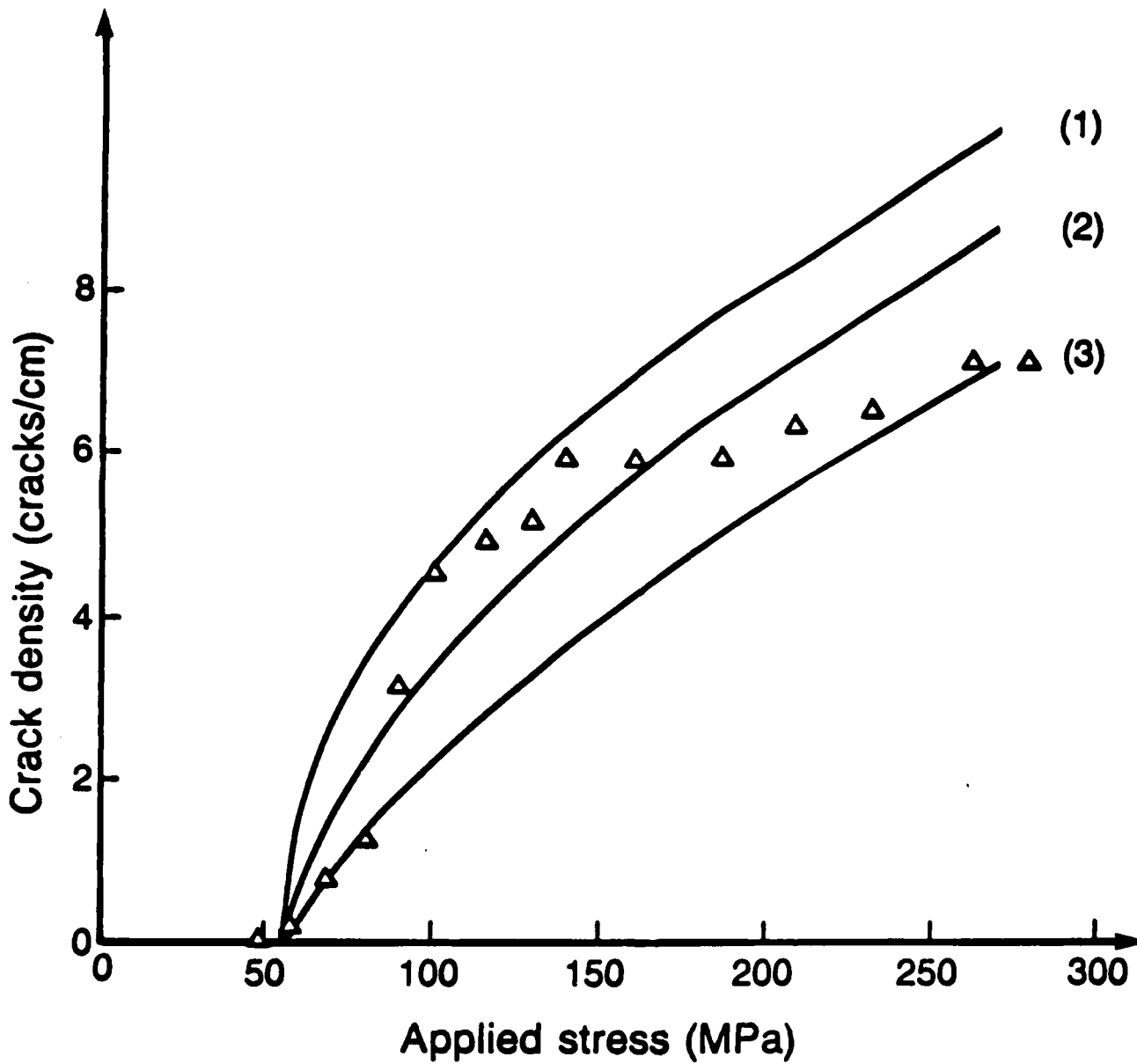


Fig. 19 Comparison of experimentally observed crack densities [25] with theoretical predictions. Curves (1), (2) and (3) corresponds to the so-numbered probability densities in the text. Here  $\lambda d = 0.8$ .

$G_r/E_p (0_2, 90_2)s$

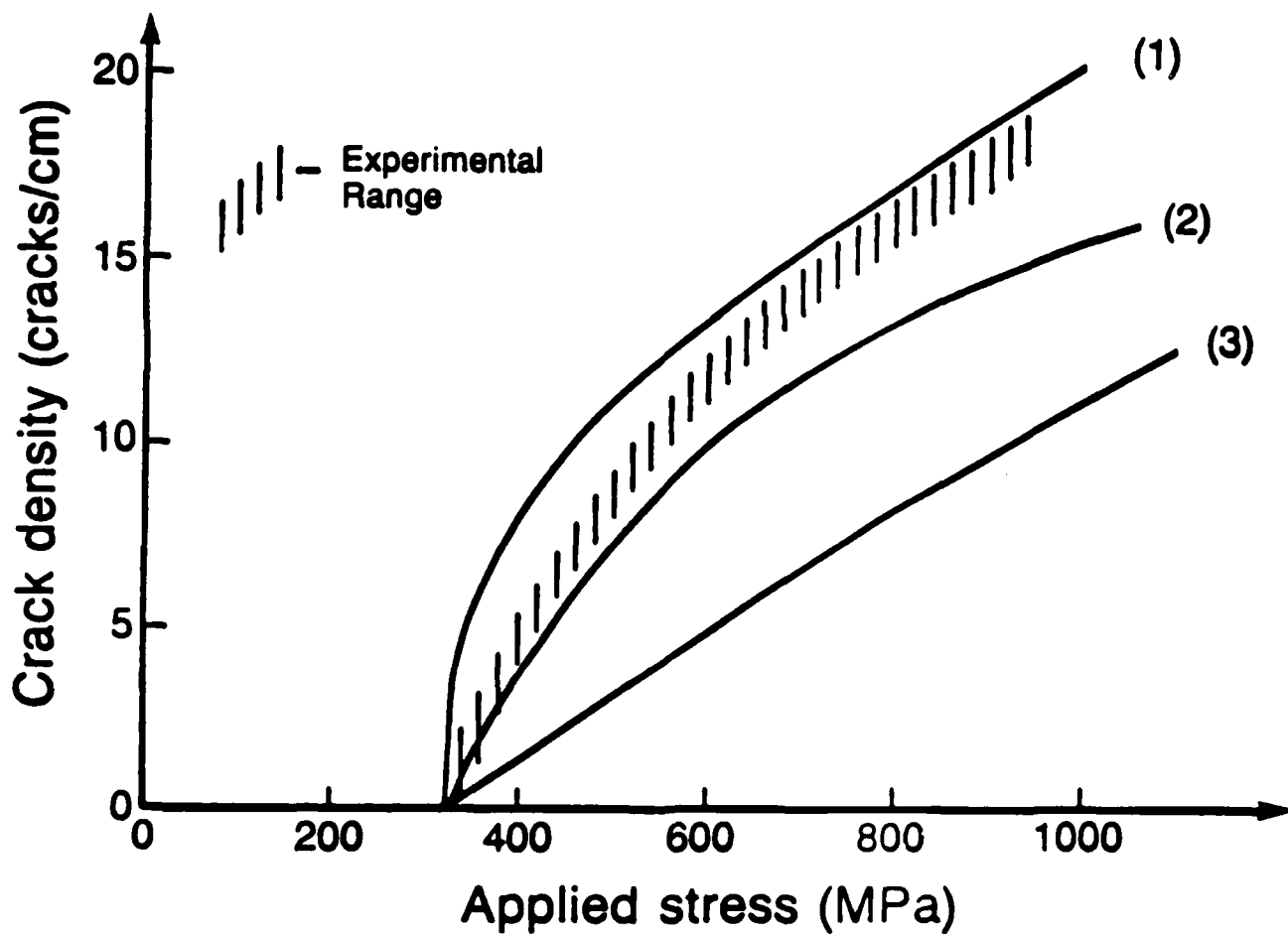


Fig. 20 Comparison of experimentally observed crack densities with theoretical predictions. Curves (1), (2) and (3) corresponds to the co-numbered probability densities in the text. Here  $\lambda_d = 1.2$ .

$Gr/E_p (0_2, 90_3)s$

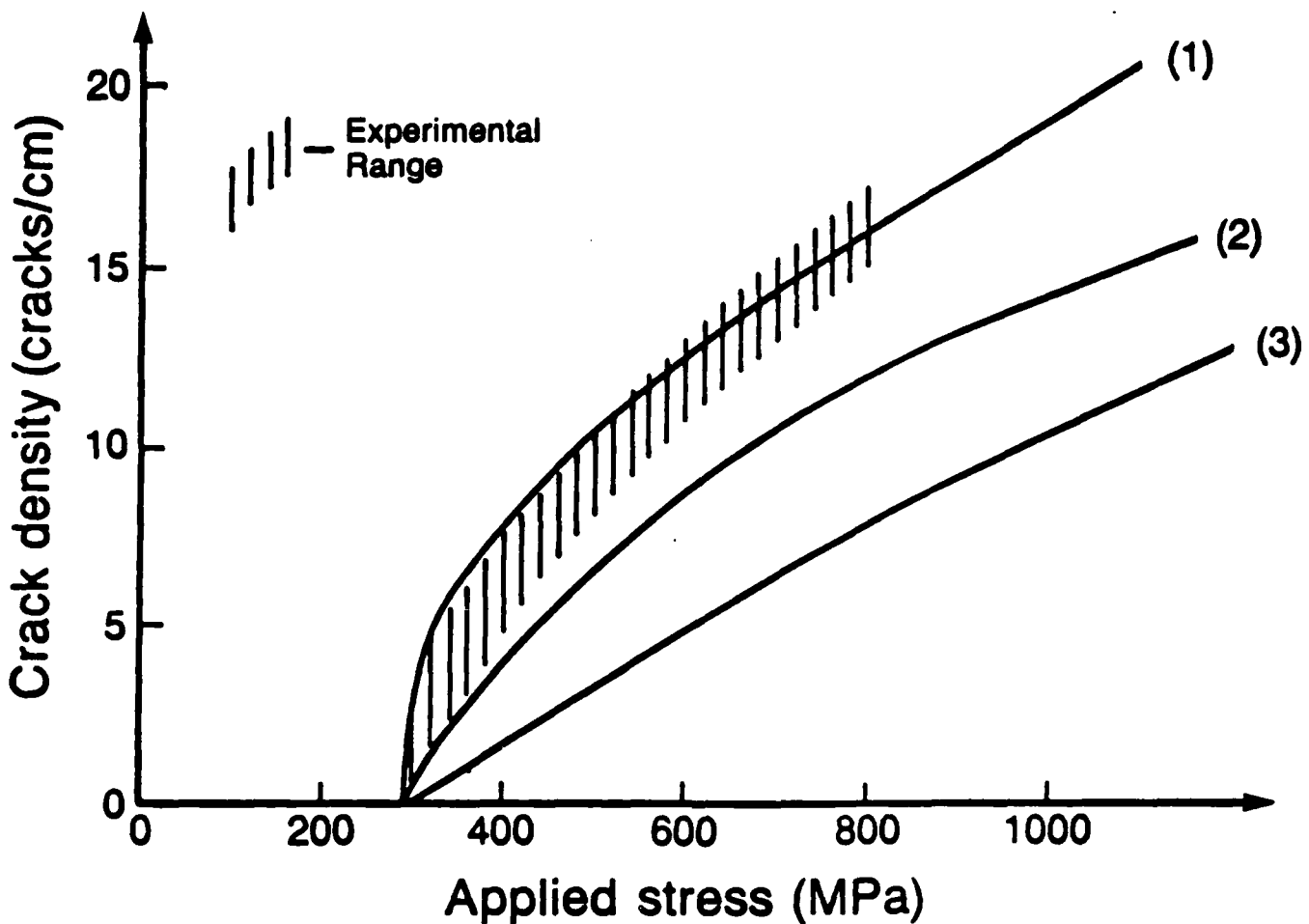


Fig. 21 Comparison of experimentally observed crack densities with theoretical predictions. Curves (1), (2) and (3) corresponds to the co-numbered probability densities in the text. Here  $\lambda d = 1.4$ .

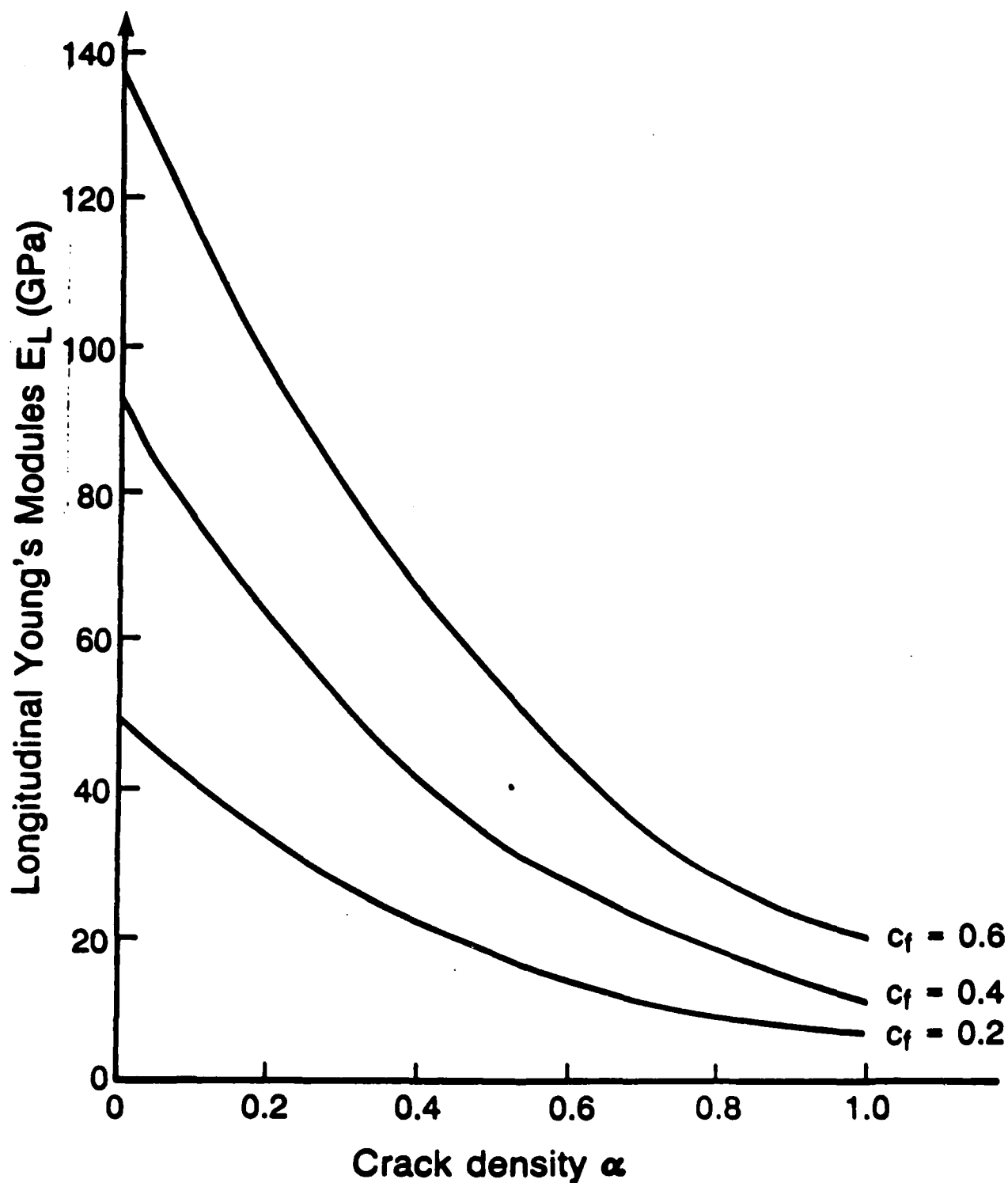


Fig. 22 Loss of longitudinal Young's modulus  $E_L$  due to fiber breaks in a unidirectional graphite epoxy composite at the indicated values of volume fraction ( $c_f$ ) of fibers

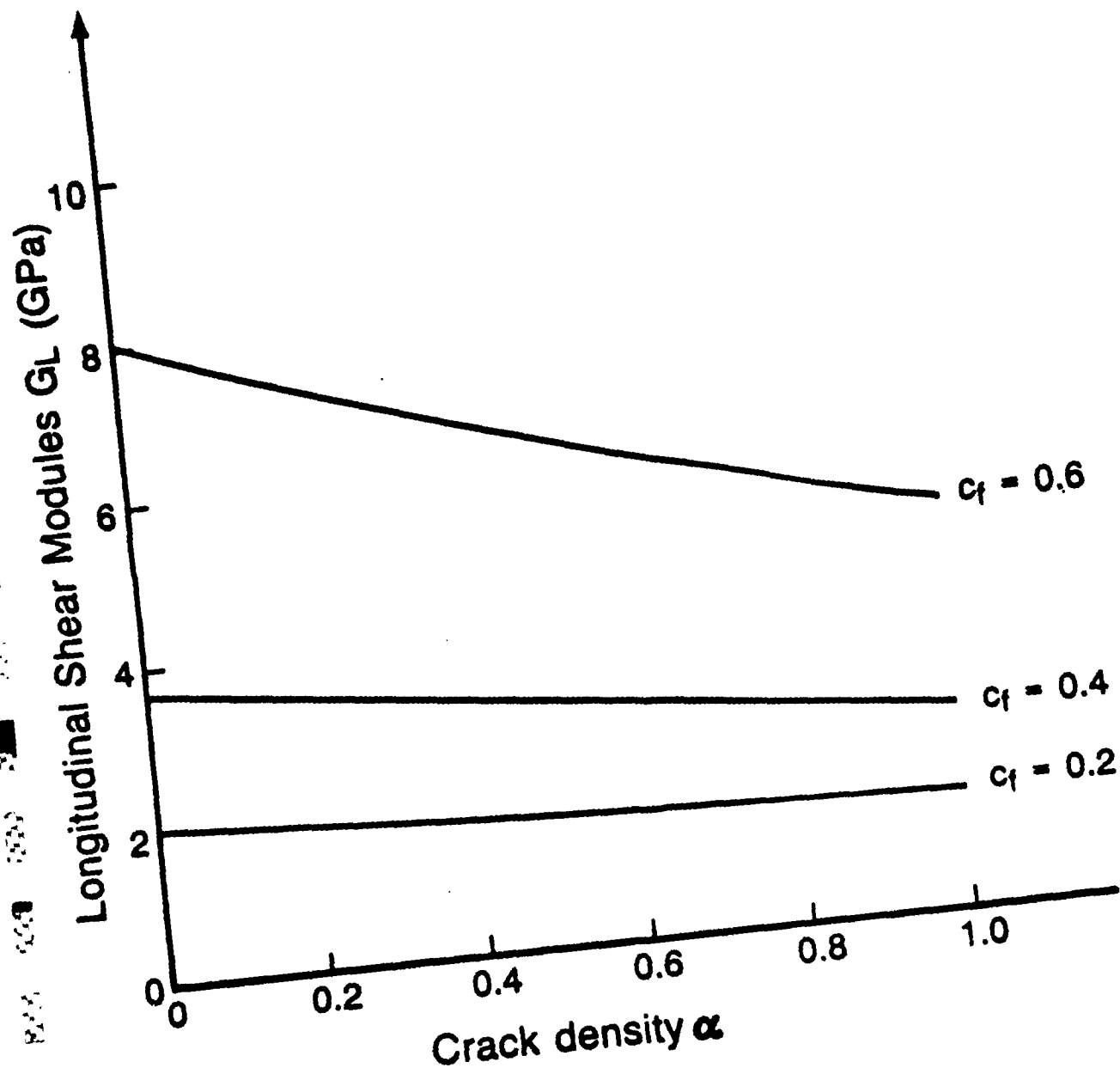


Fig. 23 Loss of longitudinal shear modulus  $G_L$  due to fiber breaks in a unidirectional graphite epoxy composite at the indicated values of volume fraction ( $c_f$ ) of fibers

$G_r/E_p (0, 90)s$

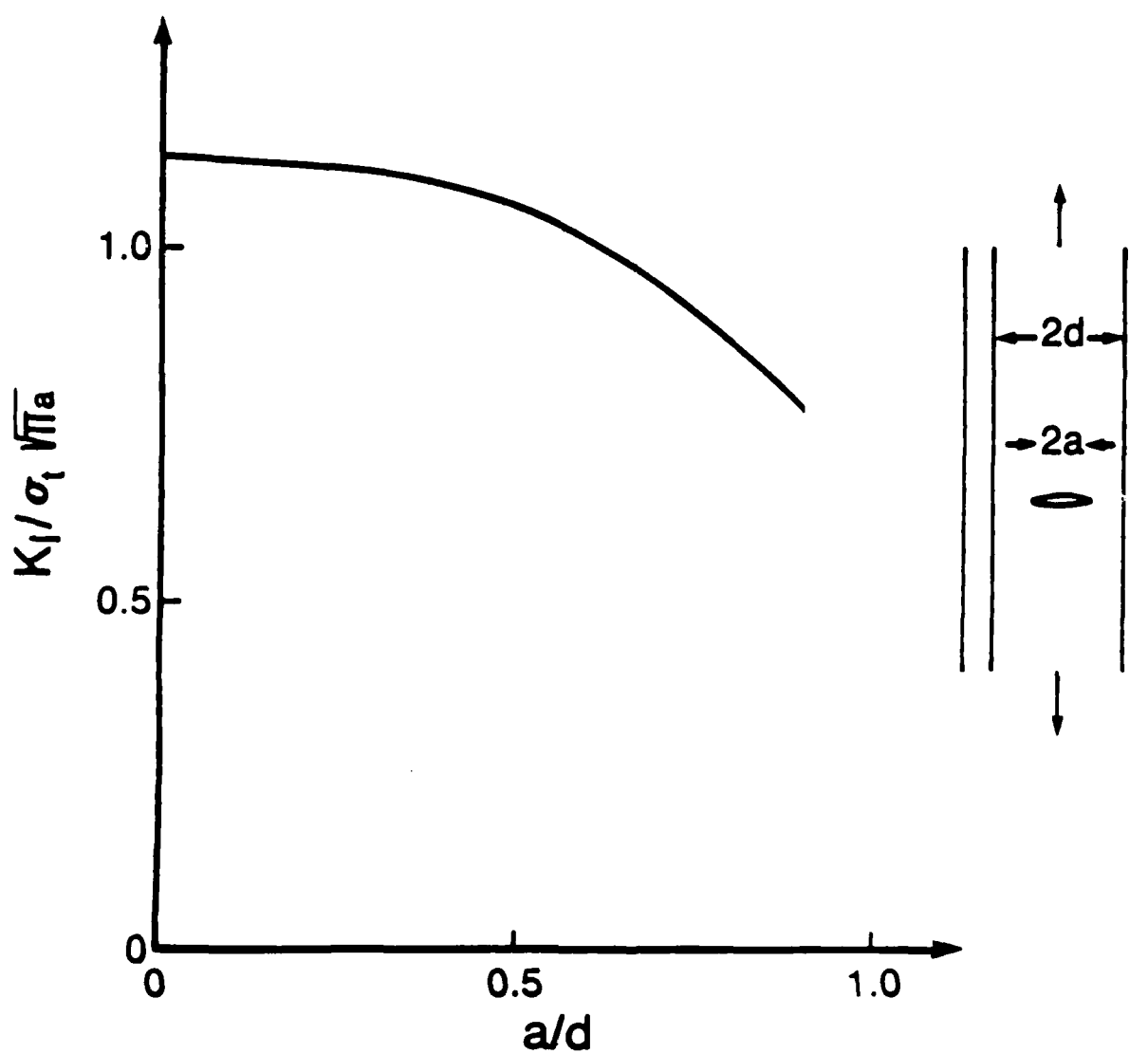


Fig. 24 Normalized stress intensity factor as a function of crack length for a plate with free lateral surfaces-plane strain

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